When two-part tariffs are not enough: Mixing with nonlinear pricing*

Steffen H. Hoernig
Universidade Nova de Lisboa and CEPR

Tommaso M. Valletti†
Imperial College London, University of Rome and CEPR

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Abstract

We consider competition in nonlinear tariffs when consumers mix two goods, and ask whether simple two-part tariffs or exclusivity can arise in equilibrium. Contrary to the existing literature, this happens only when consumer types are observable. If they are unobservable, then the equilibrium tariff has decreasing marginal prices even when goods are almost homogeneous, and a third of consumers always mixes goods. Two-part tariffs will never even arise as best responses to arbitrary tariffs.

Keywords: Mixing goods, Nonlinear tariffs, Two-part tariffs, Exclusivity

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†Corresponding author. Imperial College Business School, South Kensington campus, London SW7 2AZ, UK, email: t.valletti@imperial.ac.uk, web: www.imperial.ac.uk/people/t.valletti

1 Introduction

Nonlinear pricing with competition typically studies when customers buy exclusively from one firm. While the assumption of exclusivity fits well with many industries, it is also easy to find examples where nonlinear pricing in the form of quantity discounts coexists with buyers purchasing from multiple suppliers. Examples abound of markets where customers may enjoy combining various products (e.g., mixing different coffee beans) to create an ideal variety, yet each component of the mix may be sold under nonlinear pricing. Clearly, consumers who enjoy mixing most will have higher incentives to buy from both sources (two-stop shopping), while customers who have less to gain from mixing will buy exclusively from one firm (one-stop or exclusive shopping). The decision to buy exclusively or from both suppliers is also affected by the form of the pricing schedules offered by competing suppliers. Quantity discounts, i.e., tariffs with decreasing marginal prices, should make exclusive purchasing more likely.

We analyze a setting where, if exclusivity arises, goods are substitutes and a customer will tend to buy from the firm which is closest in the preference space if products are offered at the same price. However, if a customer finds it worthwhile to mix, then goods are complements in the sense that buying from both suppliers in variable proportions allows customers to construct ideal combinations through mixing.

Our study is related to a recent literature on nonlinear pricing under oligopoly (see Stole, 2007, for a survey). Under some assumptions, there the rather striking result arises that equilibrium competitive nonlinear tariffs can take a very simple form, namely two-part tariffs where the variable price is equal to marginal cost. The assumptions needed to reach this result typically are the following: a) a "covered" market (every consumer buys from some firm), b) no interaction between "vertical" consumer types and "horizontal" preferences, and c) symmetric costs (see Armstrong and Vickers, 2001; and Rochet and Stole, 2002). In this literature, exclusivity or one-stop shopping is assumed, rather than left to occur endogenously. As noted by Armstrong and Vickers (2001, p. 580), it is necessary to investigate more settings which are not constrained by the assumption of one-stop shopping. This paper makes a further step to fill this gap: in our model, whether individual consumers buy at both firms or buy exclusively from only one firm is determined as an equilibrium outcome of the game.

A related literature on mixed bundling allows instead customers to buy from multiple sources. Matutes and Regibeau (1992) is a seminal contribution in this area which shows that mixed bundling decreases firms' profits compared to when products are sold separately. Thanassoulis (2007) also studies mixed bundling and derives contrasting results when consumers have firm-specific preferences as opposed to product-specific preferences. However, this literature is rather extreme in that it often assumes all-or-nothing purchases. Each consumer wishes to buy only one unit of a given product, which makes it unsuitable to study nonlinear pricing (although there is nonlinear pricing in the sense of bundling discounts).

In a recent contribution, Armstrong and Vickers (2010) do allow for combining several goods and show that two-part tariffs with marginal cost pricing still arise. They illustrate that, for a given customer, if that customer type was observable, a two-part tariff would be offered as this is the pricing structure that maximizes the joint surplus between the supplier and the customer. They also show, rather intriguingly, that the fixed components (and discount if there is bundling) are independent of customer type. Thus, the same two-part tariff also can arise in equilibrium if consumer types cannot be observed. However, in Armstrong and Vickers (2010) there is no 'real' mixing, in the sense that goods are identical when consumed from either firm, and advantages or disadvantages from mixing or buying exclusively only arise from (lump sum) shopping costs and are not related to volumes.

As mentioned above, in our model with mixing, one- or two-stop shopping arises endogenously and depends on consumer type. We study a simple and very tractable Hotelling setting where we can compute the equilibrium nonlinear tariffs explicitly. The problem of combinable goods was first introduced by Anderson and Neven (1989) under simple linear pricing. Hoernig and Valletti (2007) extended it to two-part tariffs and to pure flat fees, which are pricing structures particularly relevant in media markets, where audiences may enjoy mixing among different genres offered by alternative broadcasters.[1] In this paper we take on the fully nonlinear case, which we believe to be as practically relevant as the two-part tariff case. Besides media applications, mobile telephony with penetration rates above 100% provides an example where operators offer highly nonlinear tariffs and customers may decide to buy only from one operator or to hold several SIM cards. As a further example, credit card issuers may offer discounts which grow with expenditure, yet many customers hold multiple credit cards.[2]

[1] See also Gabszewicz et al. (2004) and Gal-Or and Dukes (2003). Recently, Anderson and Gans (2009) also employ a mixing model for a broadcasting application, where it is the provider, rather than the consumer, that determines the mix (content type, i.e., politics or entertainment, in their application).

[2] See Rysman (2007) for a discussion of credit card usage. Another empirical application of nonlinear pricing under oligopoly, and where some consumers may mix, is specialty coffee, studied in McManus (2007). In an earlier contribution, Ivaldi and Martimort (1994) study competition through supply schedules in an oligopoly where two suppliers

We find that, when consumer types are perfectly observable, once again two-part tariffs can emerge for every type. Firms can simply design an efficient tariff to realize the highest possible surplus with every mixing customer, and then extract part of that surplus using the fixed component of the two-part tariff. This fixed component is type-dependent, thus when customers' types are not observed, we would expect a different equilibrium to emerge. Indeed, we find that when firms compete over fully nonlinear prices and consumer types are not observed, the equilibrium pricing structures are rather different. Two-part tariffs *never* emerge, not even in the limiting case when the market becomes perfectly competitive. Instead, equilibrium nonlinear prices always exhibit quantity discounts.

A second question that we consider is whether exclusive consumption can arise endogenously in equilibrium. In the case of observable consumer types, there are equilibria where some consumers buy exclusively if firms' marginal costs are positive, and exclusivity of all consumers can arise if marginal costs are high enough. This is in stark contrast with the case of unobservable types, where we show that full exclusivity can never arise in equilibrium.

The driver for our results is that the type of a consumer determines his mixing gains, and therefore also the quantity that he would like to buy from each firm. Thus, in our model, product preferences are a function of location, in contrast with the separability assumption in the existing literature on competitive nonlinear pricing. While, in general, the problem of nonlinear pricing when "horizontal" and "vertical" preferences interact is very complex, we can provide a fully tractable version by employing the same assumption of Anderson and Neven (1989) that the total quantity that each consumer buys is fixed, while consumers vary in how they divide this total quantity between firms.

Our paper is also clearly related to another stream in the literature, on common agency under complete or incomplete information. When applied to buyer-seller relationships, this literature typically studies two principals that sell non-homogeneous goods to the same common agent, using nonlinear pricing schedules, and investigates in particular the effects of exclusionary contracts. Previous work has found that efficient equilibria arise when there is complete information and each principal offers "truthful" schedules (Bernheim and Whinston, 1986a; Chiesa and Denicolò, 2009). While we confirm

of differentiated goods do not know the consumers' valuations for the two goods. They model this situation as a multiprincipals game where the suppliers are the principals and the consumers are the agents. Ivaldi and Martimort apply this theoretical model to study energy supply to the French dairy industry. One supplier is the public sector monopoly on electricity, while the second supplier consists of oil firms. Clearly, some buyers may want to have both sources of energy supply.

this result, we also show how, in our case, inefficient equilibria can arise under simple circumstances. Under asymmetric information, the situation is more complicated since schedules not only serve to share the rent in the bilateral coalition formed by one principal with the agent - taking as given the schedule of the rival principal - but are also used as screening devices. Independently from our work, Martimort and Stole (2009a) and Calzolari and Denicolò (2009) have recently considered (what can be interpreted as) competition by firms producing heterogenous goods for consumers. While the former paper also deals with mathematical aspects of using mechanism design techniques, both papers postulate and confirm equilibria in quadratic tariffs for a specific model, which we also find. Although our work is less general than theirs in many respects, our analysis has the added value of being able to compute explicitly fully nonlinear prices, and contrast them with explicit expressions for equilibria in linear and two-part tariffs. We find that the equilibrium in nonlinear tariffs is more efficient than the one under two-part tariffs, but less efficient than under linear tariffs.

Besides our main contribution to the literature on nonlinear pricing, a final theme that emerges in this paper relates to how the assumption of exclusivity interacts with that of observability of consumer types. We show that, when types are observable, consumers would benefit from exclusivity compared to two-stop shopping. While exclusivity destroys any gains from mixing, it also intensifies competition to the benefit of consumers. However, when consumer types are not observable and firms compete in nonlinear pricing schedules, the result is reversed: Consumers are better off with two-stop shopping (possibly buying exclusively as an equilibrium outcome). Since firms also benefit from two-stop shopping, mixing with nonlinear pricing constitutes a Pareto improvement compared to exclusivity when types are not observable. The corresponding gain in total welfare arises from the presence of mixing for some, but not all, consumers.

The rest of the paper is structured as follows. In Section 2 we set out the model. In Section 3 we consider the benchmark case where customer types are perfectly observed by firms. Section 4 considers the more realistic case where customer types are unobservable and firms must screen their customers. Section 5 concludes.

2 Assumptions about Firms and Consumers

There are two firms $i = 1, 2$, located along a Hotelling unit line at the endpoints 0 and 1. Each firm incurs a constant marginal cost $c \geq 0$ per unit supplied.

Consumers consume a total quantity normalized to 1 and are distributed uniformly between locations 0 and 1. They can decide whether to buy only from firm 1, only from firm 2, or to combine products to obtain a mix of their characteristics. Consumers can buy more than they will actually consume afterwards, i.e., there is free disposal. More precisely, if a consumer buys the quantities $\bar{q}_1$ and $\bar{q}_2$ from firms 1 and 2, respectively, with $\bar{q}_1, \bar{q}_2 \geq 0$, then he can consume some quantities q_1 and q_2 with $0 \leq q_i \leq \bar{q}_i$ for $i = 1, 2$. Firm i only observes the quantity purchased from itself, but does not observe the quantity purchased from firm j, nor does it observe how much of both goods a consumer actually consumes. We believe that this assumption realistically captures the information retailers have about their customers. As a result, neither exclusive dealing, quantity forcing nor market share contracts are enforceable: A consumer can always buy the required quantity, consume less, and then buy more units from the other firm. Thus our model is a case of "delegated common agency" as defined by Bernheim and Whinston (1986a) and analyzed in more detail by Martimort and Stole (2009a).[3]

As in Anderson and Neven (1989), a consumer located at x who combines the two products, with an amount $0 \leq q \leq 1$ of product 1 and an amount $(1-q)$ of product 2, incurs a quadratic transport cost equal to $t\,(1-q-x)^2$, where t is the unit transportation cost.[4] Consumers also derive a fixed utility v from buying from any firm, which is assumed to be high enough relative to t such that the market is always "covered" in equilibrium. This fixed utility component represents the utility of simply being able to consume, while the gain in utility due to more variety under mixing is represented by the transport cost specification.

The type of a consumer is therefore given by his location in preference space and transport costs interact with quantities consumed. The important difference to other models in the literature is that the consumer's type determines the quantities that he would buy to produce his ideal mixture. More precisely, a consumer at location x can make transport costs disappear by consuming $q = 1 - x$ from firm 1, while the transport cost would be tx^2 if he were to buy only from firm 1. In an efficient allocation, mixing should

[3] The terms "delegated" and "intrinsic" common agency refer to situations where the agent has the freedom, or not, respectively, to contract with only one of the principals. Calzolari and Denicolò (2009) refined these definitions, and our model corresponds to their notion of "private (delegated) common agency".

[4] In our model, an ideal mix can be created from existing distant products by combining them, while in the standard Hotelling model an ideal mix never arises as the customer pays no transport cost only when his ideal variety coincides with the firm's location. There the transport cost that arises if a customer purchases λ and $(1-\lambda)$ units from different firms is $\lambda tx^2 + (1-\lambda)\,t(1-x)^2$ if transport cost is proportional to quantity, and $tx^2 + t(1-x)^2$ if transport costs is lump-sum.

occur everywhere to minimize transportation costs. Whether this will actually happen in equilibrium depends on the prices charged by firms, that is, the consumer selects his optimal mix through the trade-off between reducing transport cost and reducing expenditure.

While Anderson and Neven (1989) and Hoernig and Valletti (2007) focused on competition in linear and two-part tariffs, respectively, here we consider competition in fully nonlinear tariffs. The latter can generically be stated as arbitrary functions $T_i : [0,1] \to \mathbb{R}$, $i = 1, 2$. We will later impose technical conditions on these tariffs as needed.

As a first benchmark, let us consider the case of a single monopolist selling both goods. If he offers the tariff $T(q) = vq$ for both goods, independently of whether consumer types are observable or not, the consumer at x will select the allocation that solves

$$\max_q \left\{ v - t(1-q-x)^2 - vq - v(1-q) \right\}. \tag{1}$$

The resulting quantity of the product at the endpoint 0 is $q = 1-x$, while the quantity of the product at the opposite endpoint 1 is $q = x$, i.e., the efficient allocations arise, and the corresponding consumer surplus is zero. That is, the monopolist extracts all surplus of each consumer and therefore has no incentives to distort allocations in order to extract more rents. This result can also be understood by noting that the demands of the products at the endpoints are perfectly negatively correlated when the consumers are allowed to do their optimal mixing (i.e., allow them to construct their individually optimal "bundle", without imposing it), and the monopolist exploits this.

In the following we proceed in two steps. First we consider the case where consumer types are observable and show that two-part tariffs can implement the efficient outcome, while equilibria with inefficient exclusivity can arise if marginal cost is strictly positive. In a second step, we assume that consumer types are unobservable and derive the corresponding unique differentiable equilibrium tariff. Contrary to Proposition 5 in Armstrong and Vickers (2001) and Proposition 6 in Rochet and Stole (2002), the tariffs under unobservable types are rather different from the tariffs under observability. Thus the following analysis of observable types should be understood principally as a benchmark exercise, while our main results concern unobservable types in Section 4.

3 Observable Consumer Types

If firms can perfectly observe consumer types, designing (nonlinear) tariffs becomes an exercise of first-degree price discrimination, where firms compete for selling to individual consumers. Thus we can concentrate on some generic consumer at location $x \in [0,1]$. Still, under our assumption of free disposal, firms cannot at the outset impose exclusive consumption of their good through "forcing contracts". Thus exclusivity only arises as one possible consumer choice among other mixing choices and is equivalent to a consumer buying exactly the quantity of 1 unit from one of the two firms.

Profits of selling to the consumer at x are maximized by designing a tariff such that maximum social surplus net of payments to the other firm is first created and then extracted. A first question to be answered is whether in this case Nash equilibria in nonlinear tariffs can result in the efficient allocation, and how this allocation can be implemented. The following result shows that, similar to other modeling frameworks, the efficient allocation can be implemented by a Nash equilibrium where both firms choose two-part tariffs. The proof also shows that the same allocations may be implemented using other tariffs.

Proposition 1 *If firms can observe consumer types, among the Nash equilibria in nonlinear tariffs there is a unique equilibrium in two-part tariffs. Both firms set their marginal price equal to marginal cost and fixed fees equal to each firm's marginal contribution to surplus. These tariffs implement the efficient allocation.*

Proof. Take a consumer located at x and the two-part tariff $T_2(q) = T_{20} + p_2 q$ of firm 2. Since the consumer will buy $q_2 = 1 - q_1$ from firm 2, the net surplus available to the consumer and firm 1 is given by

$$v - cq_1 - t(1 - q_1 - x)^2 - (T_{20} + p_2(1 - q_1)).$$

This is maximized at $\tilde{q}_1 = 1 - x + (p_2 - c)/2t$. Among many other nonlinear tariffs, the latter allocation can be implemented by a two-part tariff $T_1(q) = T_{10} + p_1 q$ with unique marginal price $p_1 = c$, as then the consumer solves

$$v - t(1 - q_1 - x)^2 - (T_{10} + p_1 q_1) - (T_{20} + p_2(1 - q_1)),$$

with solution $\bar{q}_1 = 1 - x + (p_2 - p_1)/2t$. Since $\bar{q}_1 = \tilde{q}_1$ iff $p_1 = c$, the key conditions on the optimal tariff are that the marginal price at $\tilde{q}_1$ is c and that the tariff's curvature is such that an interior maximum exists. Clearly both are satisfied by the above two-part tariff. Since the same holds for firm 2, equilibrium two-part tariffs necessarily involve $p_1 = p_2 = c$. As a result, the consumer receives surplus $v - c - T_{10} - T_{20}$.

Consider now firms' choices of fixed fee. Firm 1 will charge a fixed fee T_{10} at the level that makes the consumer at x indifferent between accepting the two-part tariff or buying exclusively from firm 2. Since the latter yields the consumer a utility of $v - t(1-x)^2 - c - T_{20}$, firm 1 sets $T_{10} = t(1-x)^2$. In a similar manner one shows that firm 2 will set $T_{20} = tx^2$. ■

We report the proof for completeness, but Proposition 1 is simply a direct application of Bernheim and Whinston (1986b), as a two-part tariff with marginal price equal to marginal cost is a "truthful" strategy. As is common also from other contexts with observable types, e.g., Spulber (1979) for delegated common agency when firms compete in quantities or Bernheim and Whinston (1998) for generic intrinsic common agency, in the above equilibrium each firm extracts the marginal surplus that its participation creates. In our model, this contribution consists of the elimination of the disutility related to consuming exclusively from one firm, and each firm is pivotal in realizing these gains. For consumer x, the consumer surplus and welfare resulting from efficient mixing under these two-part tariffs are given by

$$U_{obs}(x) = v - c - tx^2 - t(1-x)^2, \; W_{obs}(x) = v - c. \tag{2}$$

Mixing is efficient because every consumer chooses the efficient quantity for his type, and because no consumer is excluded. We can also readily calculate individual firms' profits,

$$\Pi_{obs} = \int_0^1 \left[t(1-x)^2 + (c-c)(1-x)\right] dx = \frac{1}{3}t, \tag{3}$$

and aggregate consumer surplus,

$$CS_{obs} = \int_0^1 \left(v - c - t(1-x)^2 - tx^2\right) dx = v - c - \frac{2}{3}t,$$

with total welfare $W_{obs} = CS_{obs} + 2\Pi_{obs}$.

While the above Proposition demonstrates that there is an equilibrium in two-part tariffs, it also shows that there will always be an infinity of possible nonlinear tariffs that also constitute an equilibrium and lead to the same allocation. The reason for this fact is that the equilibrium conditions only determine the slope (and a limit on curvature) of tariffs at the equilibrium quantity.

More importantly, the efficient allocation is not the only possible allocation in Nash equilibria of this model. Assume for example that firm 2 offers a full bundle of $\bar{q}_2 = 1$ at a (fixed) price $P_2 \geq c$, i.e., it charges a "flat fee". Then while a consumer at x can opt to buy the whole bundle and mix part of it with firm 1's good, mixing may not occur at all in equilibrium. The reason is that once the consumer buys the full bundle, at any mixed allocation he "obtains" additional units from firm 2 at a private cost of zero, while firm 1 supplies additional units at private cost c. More precisely, the net surplus for firm 1 from selling to the consumer at x is

$$v - cq_1 - t\,(1 - q_1 - x)^2 - P_2,$$

which is maximized at $q_1 = 0$ if $x \geq 1 - \frac{c}{2t}$, independently of the value of P_2, with consumer surplus $U_2 = v - t\,(1-x)^2 - P_2$. While firm 1 in this case does not induce consumers close to firm 2 to mix, it still can try to make them reject firm 2's bundle and buy exclusively from itself at price P_1. This offer is rejected if $U_2 \geq U_1 = v - tx^2 - P_1$, or $P_2 \leq P_1 + t\,(2x-1)$. Since the best offer involving non-negative profits that firm 1 can make is to sell the bundle of size 1 at price $P_1 = c$, we obtain the set of equilibrium prices under exclusivity

$$\begin{aligned} c &\leq P_2 \leq c + t\,(2x-1) \\ P_1 &= P_2 - t\,(2x-1)\,. \end{aligned} \tag{4}$$

Thus both firms offer the quantity of 1 unit, but only firm 2's offer will be accepted. As in Bernheim and Whinston (1998), the effect of the losing offer is to constrain the price of the winning offer.

Clearly (4) implies that only consumers at $x \geq \max\left\{\frac{1}{2}, 1 - \frac{c}{2t}\right\}$ will buy exclusively from firm 2. If we concentrate on the Pareto-efficient equilibrium, i.e., the one with the highest payoffs for both firms, firms charge $P_1 = c$, $P_2 = c + t\,(2x-1)$, with resulting consumer surplus and welfare under exclusive dealing with firm 2:

$$U_2(x) = v - c - tx^2,\ W_2(x) = v - c - t\,(1-x)^2\,. \tag{5}$$

Similar arguments show that consumer will accept exclusive dealing with firm 1 if $x \leq \min\left\{\frac{1}{2}, \frac{c}{2t}\right\}$. These results imply that full exclusivity, i.e. all consumers buying from only one firm, is possible if and only if $c \geq t$. We have just shown the following:

Proposition 2 *If $c > 0$ there are Nash equilibria in nonlinear tariffs (actually, flat fees) where inefficient exclusive dealing arises. Full exclusivity can arise if $c \geq t$.*

This result corresponds to the findings in the basic model of Bernheim and Whinston (1998) in terms of the structure of the equilibrium set under exclusivity, but differs strongly from their result that exclusive dealing only arises when it is efficient. On the contrary, in our model exclusive dealing is always inefficient (with the obvious exception of the two consumers at $x = 0$ and $x = 1$). Our result that inefficient exclusive dealing equilibria exist is even more surprising given that "forcing contracts", i.e., contracts based on the consumption of a specific quantity, are ineffective in our setting because consumption is not observable. Rather, once a consumer has bought the quantity $q = 1$ from one firm and then receives an offer designed to make him mix, he trades off units which he already owns to units which come at a marginal price defined by the seller. Since the seller will not be interested in making a loss on these units, he may prefer not to induce mixing. Furthermore, if he is too far from the consumer at x, he cannot win offering exclusive contracts, either.

As a last point in this section we compare the outcome under endogenous exclusivity with the one that would obtain if exclusivity were exogenous. In other words, here we consider one-stop shopping with a unit purchase, which could be the result of a commitment by either firms or consumers to not allow mixing. Given that firm i charges a total price P_i for one unit, $U_1 = v - P_1 - tx^2$ and $U_2 = v - P_2 - t\,(1-x)^2$. Thus a customer at x buys from firm 1 if $v - P_1 - tx^2 \geq v - P_2 - t\,(1-x)^2$ or $P_1 \leq P_2 + t\,(1-2x)$, exactly as above. In equilibrium, each firm sells to half of the market. For $x \geq \frac{1}{2}$, the equilibria are again given by conditions (4). Thus the outcome is identical for some consumers if $c < t$, or for all consumers if $c \geq t$.

If we again concentrate on the Pareto-efficient equilibrium, for all x the losing offer has price c and the winning one has price $c+t\,|1-2x|$. Comparing with (2), we have that

$$\begin{aligned} U_1(x) &= v - c - t\,(1-x)^2 > U_{obs}(x) \text{ for } 0 < x \leq 1/2 \\ U_2(x) &= v - c - tx^2 > U_{obs}(x) \text{ for } 1/2 \leq x < 1. \end{aligned} \tag{6}$$

In other words, we obtain that consumers are better off if they can commit to deal with only one firm. This is because under exclusivity competition is for *total* demand instead of marginal units. The former leads to overbidding, similar to Matutes and Regibeau (1992) for unit demands in complementary goods, and Calzolari and Denicolò (2009) for exclusivity with unobservable consumer types. In equilibrium, firms compete more vigorously and profits are lower compared to (3):

$$\Pi_1 = \int_0^{1/2} t\,(1-2x)\,dx = \frac{1}{4}t < \frac{1}{3}t.$$

Total consumer surplus and total welfare under exogenous exclusivity are, respectively,

$$CS = 2\int_0^{1/2} \left(v - c - t\left(1-x\right)^2\right) dx = v - c - \frac{7}{12}t,$$

and

$$W = v - c - \frac{1}{12}t < v - c.$$

Evidently, total welfare is lower since the allocation with exclusivity is inefficient due to a lack of mixing.

Given these comparative results, if individual customers could commit to exclusivity, they would benefit from this commitment - despite entirely losing the possibility to combine products - as they would pay lower prices. While it is possible to think of cases where such commitment may be credible,[5] in general it does not sound very realistic. Consumers quite rarely offer firms to become their exclusive customers, particularly in the absence of hold-up problems. We argue next that, even if commitment problems are not present, two-stop shopping arises quite naturally once consumer types are not observed by competing firms. In this case, customers enjoy strictly higher utility from being able to buy from two sources as opposed to an exclusive one.

4 Unobservable Consumer Types

4.1 Pricing Equilibrium

Assume now that firms cannot observe customers' location. Firms compete in nonlinear tariffs $T_i : [0,1] \to \mathbb{R}$, $i = 1,2$. We have shown above that with observable location x, the equilibrium two-part tariff of firm 1 is $T_1(q,x) = t\left(1-x\right)^2 + cq$. In this case, the cheapest (accepted) offer is for $x = 1$. If x is unobservable, every customer of firm 1 would select this tariff. Clearly, there is then an incentive for firm 1 to push up the fixed component. So we immediately expect that customers near $x = 0$ will stop mixing (as they gain very little in any case from mixing) and buy exclusively from firm 1. Thus with unobservable locations we expect exclusivity to arise endogenously.

[5] Continuing with the energy example of footnote 2, this corresponds to the case where industrial buyers can commit by acquiring machines or appliances that work only with gas or only with electricity.

Indeed we find the following result when considering more general nonlinear tariffs:[6]

Proposition 3 *For all $t > 0$, in the unique Nash equilibrium in nonlinear tariffs that are differentiable on $q \in (0,1)$, each firm i charges*

$$T_i(q) = (c+t)\,q + \frac{t}{3}q\,(1-q)\,. \tag{7}$$

Consumers in the intervals $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$ do not mix and buy exclusively from one firm only, while consumers in the interval $\left(\frac{1}{3}, \frac{2}{3}\right)$ mix with quantity $q(x) = 2 - 3x$ from firm 1 and quantity $1 - q(x) = 3x - 1$ from firm 2. The utility of each consumer is

$$\begin{aligned} U_1(x) &= v - (c+t) - tx^2, \quad \forall\, x \in \left[0, \frac{1}{3}\right], & (8)\\ U(x) &= v - c - \frac{2}{3}t - 2tx\,(1-x)\,, \quad \forall\, x \in \left(\frac{1}{3}, \frac{2}{3}\right), & (9)\\ U_2(x) &= v - (c+t) - t\,(1-x)^2 \quad \forall\, x \in \left[\frac{2}{3}, 1\right]. \end{aligned}$$

Total consumer surplus is $CS = v - c - \frac{29}{27}t$, and each firm's total profit is $\Pi_i = \frac{14}{27}t$.

Proof. See Appendix.

The proof proceeds according to standard mechanism design techniques as used in Stole (1995), with the twist that, due to mixing, the rival firm's tariff affects not only consumers' participation but also their incentive compatibility constraints. We first characterize, for a given rival's tariff, which consumers a firm wants to serve exclusively, which ones it wants to mix and which ones it does not want to serve. Then we determine the candidate equilibrium tariffs and confirm that they indeed form an equilibrium.

Since the equilibrium prices with linear tariffs, just as in the standard Hotelling model, are[7]

$$p^H = c + t,$$

[6] We conjecture that the following tariffs form the unique Nash equilibrium in a much larger class of tariffs than just the differentiable ones. In order to avoid too much technicality here we do not attempt to find the maximal class. For the mathematical problems involved see Martimort and Stole (2009a) and Martimort and Stole (2009b), where the latter studies discontinuous allocations in public common agency models.

[7] See Anderson and Neven (1989).

we have from (7) that

$$\frac{T(q)}{q} = p^H + \frac{t}{3}(1-q),$$

i.e., the equilibrium nonlinear tariffs imply a surcharge over the linear tariff which decreases in quantity.

Exclusive customers ($q = 1$) pay exactly the Hotelling price $c + t$, as is the case with linear pricing and with all other tariffs considered in Hoernig and Valletti (2007). Mixing customers, on the other hand, pay more per unit than under linear tariffs. These are the customers that buy smaller quantities from each supplier, since they combine both goods. Precisely because each firm is pivotal for achieving the gains from mixing, firms can exploit these mixing customers by charging higher prices for intermediate quantities.

We would also like to stress that the exclusivity that arises in equilibrium is partial and not full. Indeed, full exclusivity cannot arise in equilibrium. In our modeling framework, contracts involving full exclusivity are equivalent to tariffs $T_i(0) = 0$, and $T_i(q) = P_i$ for all $q \in (0, 1]$. Because of the possibility of free disposal, no consumer can be "punished" for consuming less than $q_i = 1$ because any price above P_i for $q_i < 1$ is not enforceable. Since these "exclusivity tariffs" are differentiable on $(0, 1)$, the above Proposition shows that they do not arise in equilibrium. Summing up:

Corollary 4 *With unobservable consumer types, full exclusivity never arises as a Nash equilibrium in nonlinear tariffs.*

As a final point in this section, we note that, similar to Calzolari and Denicolò (2009), we have found equilibrium tariffs that are quadratic functions of quantities. Yet, our result is stronger than theirs, and our methodology is different: Calzolari and Denicolò make and confirm a guess for an equilibrium in quadratic functions, while we find it as the only equilibrium candidate in the much larger class of differentiable tariffs.

4.2 Why two-part tariffs cannot arise in equilibrium

In the previous section we have shown that the unique Nash equilibrium in differentiable tariffs involves fully nonlinear tariffs, thus there is no Nash equilibrium in two-part tariffs, contrary to results in Armstrong and Vickers (2001) and Rochet and Stole (2002). What is also interesting about the equilibrium tariff (7) is that the marginal price is always above c — even above $c + t$ as a matter of fact. As $t \to 0$, with goods approaching perfect substitutes, the optimal tariff still preserves its nonlinear (actually, quadratic) shape. Note that at $t = 0$ transport costs and the mixing problem disappear, and we obtain the classical Bertrand equilibrium with zero fixed fee and price equal to marginal cost. As t approaches zero the equilibrium nonlinear tariffs approach this outcome, without ever turning into two-part tariffs.

The reason why no Nash equilibria in two-part tariffs can arise in equilibrium is made clear in the following Proposition, which contains an even stronger finding:

Proposition 5 *A two-part tariff is never a best response to any rival's (differentiable) tariff.*

Proof. In the proof of Proposition 3 contained in the Appendix, we obtain from (14) and (13), that, independently of the form of T_2, the marginal price paid to firm 1 by mixing consumers is given by

$$T_1'(q(x)) = c + 2tx.$$

That is, the marginal price that the mixing consumer at x will pay changes in x and is above c for every $x > 0$. This implies that both $q(x)$ and $T_1'(q(x))$ change with x. Thus it is not possible that a two-part tariff with $T_1' = const$ will ever be a best response in the class of nonlinear tariffs. ■

While with two-part tariffs all consumers would pay the same marginal price, under profit-maximizing nonlinear pricing, as mentioned in the preceding proof, the marginal price offered by a firm is increasing with the distance from that firm (e.g., for firm 1 it is $T_1' = c + 2tx$). Firms use this schedule of marginal prices to sort consumers and ensure that the incentive-compatibility constraints are fulfilled. Indeed, we have "no distortion at the top" at $x = 0$ (even though that falls in the exclusivity region). If the marginal price on an interval $[x, x']$ were constant, the consumer at x would claim that his true location was closer to firm 2, indicating lower marginal willingness-to-pay for the good of firm 1, which would reduce firm 1's possibility for rent extraction.

In the standard Hotelling model with lump-sum transport cost and unit demands, consumers all buy the same quantity, so there is no point in sorting them. Therefore, two-part tariffs with marginal price equal to cost can arise. In our case, on the contrary, there is a trade-off between sorting and efficiency — as is typical under second-degree price discrimination with elastic demand.

4.3 Exogenous exclusivity

As we did with observable consumer types, we now compare the equilibrium characterized by Proposition 3 with the outcome if *all* customers were assumed to buy exclusively from one firm (we have shown above that this cannot arise endogenously in equilibrium). In this case, each firm charges the standard Hotelling price $p^H = c + t$ and earns the Hotelling profit

$$\pi^H = \frac{1}{2}t < \frac{14}{27}t.$$

Thus firms are better off with nonlinear pricing and mixing.

Turning now to customers, compared to Proposition 3, we find that consumers that do not mix are unaffected, while consumers in the mixing area are also strictly better off with mixing as from (8)

$$\begin{aligned} U(x) &= v - c - \frac{2}{3}t - 2tx(1-x) \\ &> U^H(x) = v - c - t - tx^2 \quad \forall\, x \in \left(\frac{1}{3}, \frac{1}{2}\right), \end{aligned} \tag{10}$$

and similarly for mixing customers in $\left(\frac{1}{2}, \frac{2}{3}\right)$. Thus customers are (individually) better off with mixing compared to exclusivity, and there is no need to try to coordinate on exclusivity or find an appropriate commitment as in the case with observable types. In fact, compared to the case with exogenous exclusivity, nonlinear pricing with endogenous mixing achieves a Pareto improvement.

Note that efficiency is not monotonic in types. There is in fact inefficient mixing everywhere but at locations 0, $\frac{1}{2}$, and 1. At the extreme locations, customers buy exclusively from their preferred supplier which is efficient as there are no gains from mixing. At the mid-point, marginal prices are identical which induces the mixing customer to mix optimally. In all other locations, consumption is inefficient due either to exclusivity or to distorted marginal prices.

Discussion and Summary

In this paper we have considered competitive pricing under various scenarios. These scenarios differed along two dimensions: consumer types could be either observable or not observable to the two competing firms, and consumers could be assumed to buy exclusively from one firm only, or eventually mix between the two firms' offers.

In Table 1 below we recapitulate the results that we have obtained in the four possible scenarios. Under each scenario, we recall the equilibrium consumer surplus $U(x)$ for each type $x \in \left[0, \frac{1}{2}\right]$, its corresponding equation number and the individual firms' profits.

	Exclusivity (one-stop shopping)	No exclusivity (two-stop shopping)
Observable types	$U = v - c - t(1-x)^2$ (6) $\Pi_i = t/4$	$U = v - c - t + 2tx(1-x)$ (2) $\Pi_i = t/3$
Unobservable types	$U = v - c - t - tx^2$ (10) $\Pi_i = t/2$	$U = v - c - t - tx^2, \ x \in [0, \frac{1}{3}]$ (8) $= v - c - \frac{2}{3}t - 2tx(1-x), \ x \in (\frac{1}{3}, \frac{1}{2}]$ $\Pi_i = 14/27t$

As discussed earlier, a possible comparison is horizontal: for a given regime of observability, we compared the impact on consumer surplus and firm's profitability of imposing exclusivity ex ante, in contrast to allowing mixing. With observable types, the equilibrium with mixing maximizes efficiency but results in softer competition than if one-stop shopping was imposed, hence there are higher profits and lower consumer surplus in equilibrium. With mixing, prices are kept relatively high because each firm knows that its product is individually pivotal to the consumer's utility from mixing. This is in tune with earlier results in the literature on bundling. By contrast, when x is unobservable, two-stop shopping constitutes a Pareto improvement compared to one-stop shopping. Two-stop shopping allows to achieve (some) benefits from mixing that would be otherwise lost. Part of these gains can be appropriated by firms, who are still each individually pivotal to realizing gains from mixing, but at the same time some information rents must be left to customers whose type x is not observed. In equilibrium, both firms and consumers gain from mixing.

Another possible comparison is vertical: for a given regime of one- or two-stop shopping, we can also compare how consumers and firms fare with and without observability of location x. When exclusivity is imposed, we find that firms are better off and consumers are worse off when types are not observed. There are no gains from mixing in either case. Without observability, we have a model of imperfect competition with unit purchase where firms compete for the marginal consumer only. With observability, on the other hand, we have a case of first degree price discrimination with competing firms, which intensifies competition. This is most evident for the consumer at $x = 1/2$. When this type is observable, there is pure Bertrand competition and he pays only c, while with unobservable types he would pay the Hotelling price $c + t$ (plus transportation cost $t/4$ in both cases). This result persists also when exclusivity is not imposed: Consumers are worse off and firms better off when types are not observable even when there is the possibility of two-stop shopping. Take again, as a simple example, the consumer at $x = 1/2$. With observability, he pays a two-part tariff $T = cq + t/4$ to each firm and mixes optimally ($q = 1/2$), thus spending $c + t/2$ in total. With no observability and nonlinear pricing, he pays (7), again mixing optimally ($q = 1/2$), but spending $c+7t/6$ in total. This higher price is due to the firms' attempts to reduce information rents by sorting consumers. This is done by giving incentives to slightly closer (and thus more valuable) consumers to not overstate their distance.

A useful contribution of this paper is that it allows for explicit computation of nonlinear schedules, which can then be compared with the expressions for linear and two-part tariffs, also explicitly computed and known from previous work. To our knowledge, this has not been done in other papers on nonlinear pricing, while we can offer a quick assessment in our setting. We thus conclude this section by contrasting our findings with the results from Anderson and Neven (1989, AN) and Hoernig and Valletti (2007, HV). In both cases consumers are free to mix and exclusivity may arise endogenously. Both papers allow two-stop shopping but differ in the pricing structures that competing firms can offer. AN restrict firms to charging a simple linear price, and they show that in equilibrium everybody mixes efficiently. In HV, instead, firms charge two-part tariffs. These introduce inefficiencies as some customers buy exclusively, but raise firms' profits from those customers who do mix. Table 2 below summarizes the main findings from all three papers.

Pricing	Mixing	Consumers	Profits	Welfare
Linear (AN)	$[0,1]$	$v-c-1.000t$	$0.5000t$	$v-c$
Two-part (HV)	$[0.382, 0.618]$	$v-c-1.072t$	$0.5172t$	$v-c-0.03715t$
Nonlinear	$[1/3, 2/3]$	$v-c-1.074t$	$0.5186t$	$v-c-0.03704t$

As compared to our previous work, profits increase with fully nonlinear prices compared to two-part tariffs. However, notice that the change in equilibrium profits and surplus turns out to be quite small,[8] even though the equilibrium tariffs themselves are quite different: The equilibrium two-part tariff in HV is

$$T(q) = \frac{7-3\sqrt{5}}{2}t + \left(c + \frac{3\sqrt{5}-5}{2}t\right)q.$$

[8] Equilibrium profits increase by less than 0.3%. Even so, this does not imply that the individual gain of switching from a two-part tariff to a fully nonlinear tariff would necessarily have to be small.

As expected, the two-part tariff lies above (7) for small values of q, and below (7) for higher values of q.[9] While the inefficiency due to too little mixing still exists under nonlinear tariffs, the number of exclusive customers is smaller than under two-part tariffs.

5 Conclusions

We have considered a model of competition over nonlinear tariffs in a market where consumers mix their ideal variety from two different combinable goods. If types are observable, there is a Nash equilibrium in two-part tariffs with an efficient market outcome. On the other hand, if types are unobservable then the unique Nash equilibrium in the class of differentiable tariffs is fully nonlinear, and inefficiencies arise due to endogenous exclusivity. This holds true even if the market becomes very competitive, which is a decisive difference to models where consumers either consume one good or the other. Indeed, the best response to any type of tariff of this class is never a two-part tariff in this model.

We have also found that, with observable locations, full exclusivity, i.e., all consumers buying from only one firm each, can arise endogenously if marginal cost is high enough, while with unobservable locations exclusivity is only partial. That is, in the latter case the consumers with the weakest preferences for either good will always mix in equilibrium.

The key driver of our results is that product preferences are a function of location, whereas the efficient two-part tariff results in the literature of competitive nonlinear pricing are for the case where (possibly heterogeneous) product preferences, as distinct from firm preferences, are independent of location. We also assumed that all consumers purchase the same quantity in total, although consumers may vary in how they divide this quantity between firms. This assumption improved tractability and should be relaxed in future work. The result that two-part tariffs do not arise in equilibrium should continue to hold with elastic total demand as long as consumers' demands for the two goods remain interdependent.

[9] The threshold value is $q = \frac{3(7-3\sqrt{5})}{2} \cong 0.438$. The two tariffs also coincide for $q = 1$, when they both amount to $c + t$.

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Appendix

Proof of Proposition 3. Assume that the nonlinear tariff T_2 is differentiable on $(0,1)$, but make no assumption about continuity or differentiability of T_2 at $q = 1$ or $q = 0$. Because of the assumption of free disposal, we can focus our attention on non-decreasing tariffs T_2: If $T_2(q') > T_2(q'')$ for some $q' < q''$ then a tariff $\tilde{T}_2$ with $\tilde{T}_2(q') = T_2(q'')$ would lead to identical consumption choices, the same revenue for firm 2, but strictly higher costs. Thus tariffs must be non-decreasing in equilibrium.

We first consider firm 1's problem of maximizing profits subject to participation and incentive compatibility constraints, keeping T_2 fixed and following closely Stole (1995). Stole assumes exclusivity, which means that an individual consumer's demand for each of the two varieties does not depend on how much he would buy of the other variety. As a result, the rival's offer only appears in firm 1's participation constraint. In our case consumers can buy from both firms simultaneously, and therefore demands for both varieties are interrelated. The rival's offer then appears in both participation and incentive compatibility constraints. In particular, this implies that the slope of the best response menu, and not only its intercept, is dependent on the rival firm's tariff. Our main technical innovation over Stole (1995) therefore consists of dealing with this additional level of complexity.

Firm 1 offers a menu $\{p(x), q(x)\}$ to consumers at $x \in [0, \bar{x}]$, where $\bar{x} \leq 1$ is firm 1's marginal consumer, i.e., $q(x) = 0$ for $x > \bar{x}$. If q is strictly decreasing in x while $q(x) \in (0,1)$, which below we show to be true, then $T_1(\lambda) = p(q^{-1}(\lambda))$. For non-mixing (exclusive) consumers we have $q(x) = 1$ and $p(x) = T_1(1)$.

Denoting $\tilde{U}(x, \hat{x}) = u(x, q(\hat{x})) - p(\hat{x})$ and $U(x) = \tilde{U}(x, x)$, firm 1 maximizes its profits subject to incentive compatibility and participation constraints:

$$\begin{aligned} \max_{p,q} \Pi_1 &= \int_0^{\bar{x}} (p(x) - cq(x))\, dx \\ s.t.\ U(x) &\geq \tilde{U}(x, \hat{x}) \ \forall x, \hat{x} \in [0, \bar{x}], \\ U(x) &\geq U_2(x) = v - T_2(1) - t(1-x)^2 \ \forall x \in [0, \bar{x}]. \end{aligned} \tag{11}$$

As usual, the incentive compatibility constraints can be expressed as

$$U(x) = U(\bar{x}) - \int_x^{\bar{x}} u_x(s, q(s))\, ds,$$

where $u_x(x, \lambda) = 2t(1 - \lambda - x)$. We have $U'(x) = 2t(1 - q(x) - x) < U_2'(x) = 2t(1-x)$ while $q(x) > 0$, i.e., $U(x) - U_2(x)$ is strictly decreasing.

Therefore the participation constraint is binding at $\bar{x}$ and slack at $x < \bar{x}$. After the substitution $p(x) = u(x, q(x)) - U(x)$ we obtain

$$\Pi_1 = \int_0^{\bar{x}} \left(u(x, q(x)) - cq(x) + \int_x^{\bar{x}} u_x(s, q(s))\, ds \right) dx - \bar{x}U(\bar{x}).$$

Furthermore,

$$\begin{aligned} & \int_0^{\bar{x}} \int_x^{\bar{x}} u_x(s, q(s))\, ds dx = \int_0^{\bar{x}} \int_x^{\bar{x}} u_x(s, q(s))\, ds * 1 dx \\ = & \left[\int_x^{\bar{x}} u_x(s, q(s))\, ds * x \right]_0^{\bar{x}} - \int_0^{\bar{x}} \left(-u_x(x, q(x)) * x \right) dx \\ = & \ 0 + \int_0^{\bar{x}} x u_x(x, q(x))\, dx. \end{aligned}$$

Thus the objective function is transformed into the simpler form

$$\Pi_1 = \int_0^{\bar{x}} \left[u(x, q(x)) - cq(x) + x u_x(x, q(x)) \right] dx - \bar{x}U(\bar{x}). \tag{12}$$

The integrand is to be maximized over $q(x) \in [0, 1]$ separately for each $x \in [0, \bar{x}]$. This step defines which consumers the firm will want to serve exclusively (or not at all) and which it wants to mix, both given the other firm's tariff. Since tariffs are non-decreasing, consumers will buy just the amount that they need, i.e. the total quantity sold is equal to the total consumed.

If firm 1 intends to sell a strictly positive quantity but not impose exclusive dealing on the consumer at x, we obtain the necessary first-order condition

$$\begin{aligned} & T_2'(1 - q(x)) + 2t(1 - q(x) - x) \qquad (13) \\ = & \ u_\lambda(x, q(x)) = c - x u_{x\lambda}(x, q(x)) = c + 2tx. \end{aligned}$$

From firm 1's point of view, the mixing client at x solves $\max_{\lambda \in (0,1)} u(x, \lambda) - T_1(\lambda)$, where

$$u(x, \lambda) = v - T_2(1 - \lambda) - t(1 - \lambda - x)^2.$$

Assume now that T_1 is differentiable on $(0, 1)$, therefore a choice of quantity $\lambda \in (0, 1)$ is characterized by the first-order condition

$$u_\lambda(x, \lambda) = T_1'(\lambda). \tag{14}$$

Here and in the following, subscripts $_x$ and $_\lambda$ denote partial derivatives.

From (14) and (13) we obtain $T_1' = u_\lambda = c + 2tx$. Deriving the corresponding identity for firm 2, the consumer at x is offered a marginal price of $T_2' = c + 2t(1-x)$. Substituting this into (13) and solving for $q(x)$, we obtain $q(x) = 2 - 3x$ on $x \in \left(\frac{1}{3}, \frac{2}{3}\right)$, $q(x) = 1$ for $x \leq \frac{1}{3}$, and $\bar{x} \leq \frac{2}{3}$. With $q^{-1}(\lambda) = \frac{2-\lambda}{3}$, we have $T'(\lambda) = c + 2t\frac{2-\lambda}{3}$ for $\lambda \in (0,1)$, equal for both firms. Allowing for fixed fees T_{10} and T_{20}, firm 1's nonlinear equilibrium candidate tariff is, for $\lambda \in (0,1)$,

$$\begin{aligned} T_1(\lambda) &= T_{10} + \int_0^\lambda \left(c + 2t\frac{2-s}{3}\right) ds = T_{10} + c\lambda + \frac{t}{3}\lambda(4-\lambda) \qquad (15) \\ &= T_{10} + (c+t)\lambda + \frac{t}{3}\lambda(1-\lambda). \end{aligned}$$

We also have $T_1(1) = \lim_{\lambda \nearrow 1} T_1(\lambda) = T_{10} + (c+t)$, because charging more would violate the marginal exclusive consumers' incentive compatibility constraints and charging less would result in lower profits.

At the above candidate tariffs, the necessary second-order conditions for the above-mentioned interior maximization problems hold because $T_i'' = -\frac{2}{3}t$: The one corresponding to (14) is $2t + T_1''(\lambda) + T_2''(1-\lambda) = \frac{2}{3}t \geq 0$, and the one corresponding to (13) is $2t + T_2'' = \frac{4}{3}t \geq 0$.

Consumer surplus at location x is

$$\begin{aligned} U_1(x) &= v - T_{10} - (c+t) - tx^2, \quad \forall\, x \in \left[0, \frac{1}{3}\right], \\ U(x) &= v - T_{10} - T_{20} - c - \frac{2}{3}t - 2tx(1-x), \quad \forall\, x \in \left(\frac{1}{3}, \frac{2}{3}\right), \\ U_2(x) &= v - T_{20} - (c+t) - t(1-x)^2 \quad \forall\, x \in \left[\frac{1}{3}, 1\right]. \end{aligned}$$

Firm 1's marginal consumer $\bar{x} \leq \frac{2}{3}$ is indifferent between mixing and buying only from firm 2 if $U(\bar{x}) = U_2(\bar{x})$, or $\bar{x} = \frac{2}{3} - \sqrt{T_{10}/3t}$. This implies $T_{10} \leq t/3$ for mixing to occur. For higher T_{10} profits will be smaller because firm 1 loses exclusive customers, and negative T_{10} is not desirable because firm 1 does not want to sell to consumers beyond $x = \frac{2}{3}$.

Substituting (15) into (12) leads to

$$
\begin{aligned}
\Pi_1 &= \int_0^{\frac{1}{3}+\sqrt{T_{20}/3t}} \left(v - c - 3tx^2\right) dx \\
&\quad + \int_{\frac{1}{3}+\sqrt{T_{20}/3t}}^{\frac{2}{3}-\sqrt{T_{10}/3t}} \left(v - c + \frac{2}{3}t - 4tx + 3tx^2 - T_{20}\right) dx \\
&\quad - \left(\frac{2}{3} - \sqrt{T_{10}/3t}\right) U \left(\frac{2}{3} - \sqrt{T_{10}/3t}\right) \\
&= \frac{1}{27}\left(14t + 9T_{20} + 3T_{20}\sqrt{3T_{20}/t} - 6T_{10}\sqrt{3T_{10}/t}\right).
\end{aligned}
$$

The maximum over $0 \leq T_{10} \leq t/3$ is at $T_{10} = 0$. Thus the equilibrium fixed fees are $T_{10} = T_{20} = 0$, and profits are $\Pi_1 = \frac{14}{27}t$.

Total consumer surplus is

$$
\begin{aligned}
CS &= 2\int_0^{\frac{1}{3}} \left(v - (c+t) - tx^2\right) dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(v - c - \frac{2}{3}t - 2tx\,(1-x)\right) dx \\
&= v - c - \frac{29}{27}t.
\end{aligned}
$$

While in the derivation of T_1 and T_2 we assumed that both were differentiable on $(0,1)$, inserting the T_2 we just derived into (13) and determining $q\,(.)$, demonstrates that T_1 is a best response to T_2 also in the space of all nonlinear contracts. Thus we have found a Nash equilibrium in generic nonlinear tariffs, which is unique within the class of differentiable tariffs. ■