Bundling Revisited: Substitute Products and Inter-Firm Discounts*

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Abstract

This paper extends the standard model of bundling to allow for substitutable products and for products to be supplied by separate sellers. Firms have an incentive to introduce bundling discounts when the demand for a bundle is elastic relative to demand for stand-alone products. We describe when an integrated monopolist who supplies substitute products wishes to offer a bundle discount. Separate firms often have a unilateral incentive to offer inter-firm bundle discounts when products are substitutes, although this depends on the detailed form of substitutability. Bundle discounts mitigate the innate substitutability of products, which can relax competition between firms.

1 Introduction

Bundling—the practice whereby consumers are offered a discount if they buy several distinct products—is used widely by firms, and is the focus of a rich economic literature. However, most of the existing literature discusses the phenomenon under relatively restrictive assumptions, namely:

- a consumer's valuation for a bundle of several products is the sum of her valuations for consuming the items in isolation, and
- bundle discounts are only offered for products sold by the same firm.

These two restrictive assumptions are related, in that when valuations are additive, it is rarely the case that a firm would wish to condition its price on whether or not its customer

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has also purchased a product from another seller. This paper analyzes the incentive to engage in bundling when one or both of these assumptions is relaxed.

We focus on the case where products are partial substitutes, in the sense that a consumer's value for a bundle is lower than the sum of stand-alone valuations. (As mentioned later, the case with complements has been studied before.) In broad terms, when linear prices are used an integrated monopolist tends to raise its price when its products become more substitutable, while separate sellers tend to lower their price when products are more substitutable. The impact of a bundle discount is to mitigate or reverse the extent of product substitutability, since the reduced price for joint consumption acts to reduce the disutility of joint consumption due to sub-additive preferences. As such, when an integrated firm engages in bundling, it may reduce *all* of its prices. When separate sellers engage in inter-firm bundling, this may induce them to raise their regular price. Indeed, when separate sellers agree on a bundle discount in advance of price competition, this could act as an instrument of collusion.

In more detail, the plan of the paper is as follows. In section 3, we present a fairly general analysis of the incentive to introduce bundling discounts, starting from a position where linear prices are offered. Both situations where products are supplied by an integrated monopolist and where separate products are supplied by separate monopolists are covered. In broad terms, there is a motive to offer bundle discounts when consumer demand for the bundle is relatively elastic compared to demand for stand-alone items.

In section 4, we specialise the framework to the case where the two products are symmetric, which makes the analysis a good deal more transparent. When valuations are additive and products are supplied by separate sellers, we show that a firm has an incentive to offer a discount when its customer also buys the other product only when valuations for the two products are negatively correlated. When products are partial substitutes, an integrated firm has an incentive to bundle whenever the proportion of those consumers who buy a product at price p and who go on to buy the other product at the same price decreases with p. In examples, it is often the case that when an integrated firm engages in bundling all its prices fall relative to the situation with linear pricing.

We analyze two special forms of product substitutability in more detail. First, when there is a constant disutility of joint consumption we show that an integrated firm wishes to engage in bundling whenever the firm would wish to do so in the parallel situation where valuations were additive. Thus, allowing for this form of substitution does not make it "less likely" that a monopolist wishes to bundle. However, the optimal bundle discount is smaller than when valuations are additive. We also show that separate sellers typically wish to offer a joint-purchase discount in this case: the fact that a customer has purchased the rival product implies that her incremental valuation for the firm's own item has fallen, and this usually implies that the firm would like to reduce its price to this customer. Second, we consider the situation in which a proportion of buyers can only consume a single item (for instance, a tourist in a city might only have time to visit a single museum), while the remaining consumers have additive valuations. Here, an integrated firm wishes to offer a bundle discount under the same conditions as when all consumers had additive valuations. However, with this alternative form of substitution, separate sellers would like, if feasible, to charge a *premium* when a customer also buys the rival product. The fact that a consumer wants to buy both products implies that she has additive valuations, and there is no competition between sellers for these consumers.

Finally, in section 5 we investigate *partial* pricing coordination between separate sellers. (The earlier parts of the paper considered the two polar cases where separate sellers did not coordinate their tariffs at all and—in the integrated firm analysis—where the two suppliers fully coordinated their tariffs.) Specifically, we suppose that firms first agree on an inter-firm discount (which they fund equally), and subsequently they choose their regular prices without coordination.[1] When valuations are additive, we show that such a scheme will usually raise each firm's profit, and, at least when valuations are independent, its operation will also boost total welfare. However, when sellers offer substitute products, the agreed bundle discount acts to reduce the effective substitutability between products, inducing firms to raise their prices. Thus, the scheme can act as an instrument of collusion.

This paper is not the first to investigate these and related issues. A number of papers have investigated whether or not "code sharing"—i.e., coordinated pricing by separately-owned airlines for multi-flight itineraries—is an efficient practice. Multi-flight itineraries are products made up of complementary components, and so the inefficiency of uncoordinated pricing by separate airlines is due to double marginalization. An early theoretical

[1] Examples of this kind of arrangement include supermarkets and gasoline stations agreed on a discount on gasoline if the customer has been to the supermarket, or credit cards offering discounts proportional to spend on designated hotels or flights. Currently, *Amazon.co.uk* offers its customers a variety of promotions (in the form of discount vouchers enclosed when books are delivered to customers) for seemingly unrelated products (such as wine) supplied by independent sellers.

contribution to this literature is Brueckner (2001), who provides a model in which two airlines need to cooperate to prevent double-marginalisation on some city-pair routes, but compete on other routes. In his model, if the two firms can coordinate their prices on all routes, the benefits of price reductions on the non-competitive routes tend to outweigh the harm done by allowing collusion on the competitive routes.

The incentive for a multiproduct monopolist to offer a discount for purchase of multiple items is discussed by Stigler (1963), Adams and Yellen (1976), Long (1984) and McAfee, McMillan, and Whinston (1989), among many others. The latter two papers showed that it is optimal to introduce a bundle discount when the distribution of valuations is statistically independent and valuations are additive, suggesting that a degree of joint pricing is optimal even for unrelated products. Except for Long, these papers assume that valuations are additive. Long's paper proposed an intuitive "economic" analysis of the incentive to bundle, which is adopted to a large extent in the current paper. Long's approach is discussed in detail in section 2.

The early literature on bundling also included papers by Schmalensee (1982) and Lewbel (1985), who studied the incentive for a single-product monopolist unilaterally to offer a bundling discount if its customers also purchased a competitively-supplied product. Since the two products can be independent (or even substitutes) in their analysis, their argument is distinct from the idea that tying a monopoly product with a competitively-supplied *complementary* product can be used as a "metering" device to facilitate price discrimination. Consider Schmalensee's argument in more detail. There are two items for sale to a population of consumers, and item A is available at marginal cost due to competitive pressure, while item B is supplied by a monopolist. Valuations are additive, and the fact that a consumer buys item A has no impact on her valuation for item B. The items are not independent in the statistical sense, though, and that fact that a consumer is willing to buy item A is informative to the monopolist. If there is negative correlation between valuations for the two items, the fact that a consumer buys item A is "bad news" for the monopolist, who then has an incentive to set a lower price to its customers who also buy A. Lewbel extends this analysis, to allow the two items to be partial substitutes. In this case, the fact that a consumer buys item A is also bad news for the monopolist, and provides an additional reason for it to offer a bundling discount for joint consumption.

Bundling arrangements between separate firms have been analyzed by Gans and King (2006), who investigate a model with two kinds of products (gasoline and food, say), and each kind of product is supplied by two differentiated firms. When all four products are supplied by four separate firms and firms set their prices independently, there is no interaction between the two kinds of product. However, two firms (one offering each of the two kinds of product) can enter into an alliance and agree to offer consumers a discount if they buy both products from the alliance. (In the model, the joint pricing mechanism is similar to the one used in section 5 below: the firms decide on their bundle discount, which they agree to fund equally, and then they set their prices non-cooperatively.) Gans and King observe that when a bundle discount is offered for joint purchase of otherwise independent products, those products are then converted into "complements". In their model, in which consumer tastes are uniformly distributed, a pair of firms does have an incentive to enter into such an alliance, but when both pairs of firms do this, their equilibrium profits are unchanged from the situation when all four firms set independent prices, although welfare and consumer surplus fall. Indeed, the equilibrium discounts are so large that all consumers buy both products from one alliance or the other, so there is "pure bundling".

There is a substantial literature on "meet the competition" offers by firms, whereby a retailer offers to refund the difference (or more than the difference) if a customer documents a lower price for the same item at a different store. (See Salop (1986) for early discussion of this practice.) In effect, such a policy conditions a firm's price on rival *prices*, while in the current paper we suppose that a firm can condition its price only on whether a consumer also buys from another firm. Because price-matching guarantees can blunt incentives to undercut rivals, this apparently pro-consumer policy can act as an instrument of collusion, just as agreements to offer bundle discounts do in the current paper.

Finally, Lucarelli, Nicholson, and Song (2010) discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set two different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS in the early 2000s set very different prices for similar chemicals depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails which combine drugs from different firms.

Although in this market firms do not price drugs differently depending whether the drug is used in cocktail (unlike the HIV/AIDS market), they estimate the impact when one firm engages in this form of price discrimination. They find that a firm will typically (but not always) reduce the price for stand-alone use and raise the price for bundled use.

2 An Economic Model of Bundling

In a clever note, Long (1984) presents what could be termed an "economic" model of bundling. Rather than focussing on a diagrammatic exposition concentrating on the details of joint distributions of two-dimensional consumer valuations, he uses standard tools from demand theory to derive conditions under which a bundling discount is optimal. Here, I recapitulate his analysis in its simplest, symmetric form. (Long also analyzes the situation where products are asymmetric.)

Suppose there are two symmetric products supplied by an integrated monopolist, labelled 1 and 2, each of which has constant marginal cost c. A consumer wishes to buy either zero or one unit of each product (and may wish to buy a unit of both products). Due to the assumed symmetry of demand and cost, suppose the firm sets the same price p for buying a unit of either product. Potentially, the firm also offers a discount δ if the consumer buys both products, so that the total price for buying both products is $2p - \delta$. Write the proportion of all potential consumers who buy just one item as X_1 and the proportion who buy both items as X_2. The firm's profit is therefore

$$\pi = (p - c)X_1 + (2p - \delta - 2c)X_2 \ ,$$

which can be re-written as

$$\pi = \delta N + (P - c)X \ , \tag{1}$$

where $N \equiv X_1 + X_2$ is the proportion of consumers who buy something, $X \equiv X_1 + 2X_2$ is the total number of units supplied, and $P = p - \delta$ is the incremental price of one product given the consumer buys the other product. Thus, (1) shows how the bundling tariff can be viewed as a two-part tariff comprising a fixed charge δ and marginal price P. Viewing the two demands N and X as functions of δ and P, standard demand theory indicates that cross-price effects are symmetric, so that $N_P \equiv X_\delta$ (where subscripts denote partial derivatives).

The question of whether it is optimal for the monopolist to introduce a bundling discount is therefore equivalent to whether it is optimal to have a positive fixed charge in the two-part tariff. Let P^* be the monopolist's most profitable price when no bundle discount is offered, i.e., P^* maximizes $(P-c)X(0,P)$. Starting from this situation with linear pricing, consider the impact on profit of introducing a small discount $\delta > 0$, keeping the marginal price fixed at P^*. From (1), the impact on profit has the sign of

$$\left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} = N + (P^* - c)X_\delta = N + (P^* - c)N_P = N - \frac{X}{X_P}N_P \overset{sign}{=} -\frac{\partial}{\partial P}\frac{X}{N} .$$

Here, the third equality follows from the first-order condition for the optimality of P^*. Thus, introducing a bundle discount raises profits if average demand per consumer, X/N, falls with price (in the neighborhood of the optimal price P^*). Since

$$\frac{X}{N} = \frac{X_1 + 2X_2}{X_1 + X_2} = 1 + \frac{X_2/X_1}{1 + X_2/X_1} ,$$

the condition requires that the ratio X_2/X_1 decreases with price, so that demand for a single item is less elastic than demand for the bundle. (This discussion presumes that there is some two-item demand, so that $X_2 > 0$.) We summarize this result as:

Result (Long, 1984): *Suppose an integrated monopolist supplies two symmetric products. The firm has an incentive to introduce a discount for buying the bundle whenever the elasticity of demand for buying a single item (as the linear price p varies) is lower than the elasticity of demand for buying both items (as the linear price p varies).*

In economic terms, this elasticity condition is intuitive. If the firm initially charges the same price for buying a single item as for buying a second item, and if demand for the latter is more elastic than demand for the former, then the firm would like to reduce its price for buying a second item.

Consider the knife-edge case where a consumer's value for the bundle is simply the sum of her individual stand-alone values. That is, the stand-alone value for product $i = 1, 2$ is v_i and her value for the bundle is $v_1 + v_2$. With additive values, if the firm offers the linear price p for buying either item the consumer's buying decision is simple: she should buy product i whenever $v_i \geq p$, as shown on Figure 1. Suppose that the marginal c.d.f. for either value v_i is $F(v_i)$. A useful way to capture the extent of correlation in values is the function

$$\Psi(p) \equiv \Pr\{v_2 \geq p \mid v_1 \geq p\} \ . \tag{2}$$

Then, as shown on the figure, the proportion of consumers who buy both items with linear price p is $X_2 = (1 - F(p))\Psi(p)$, while the fraction who buy any single item i is $(1 - F(p))(1 - \Psi(p))$, so that $X_1 = 2(1 - F(p))(1 - \Psi(p))$. It follows that X_1/X_2 increases with p whenever

$$\Psi(p) \text{ is strictly decreasing in } p \ . \tag{3}$$

Clearly, condition (3) holds if v_1 and v_2 are independently distributed, but it also applies much more widely. Indeed, the beauty of Long's approach is that his broad framework applies just as well to situations in which valuations are not additive, as we discuss in more detail in the following analysis.

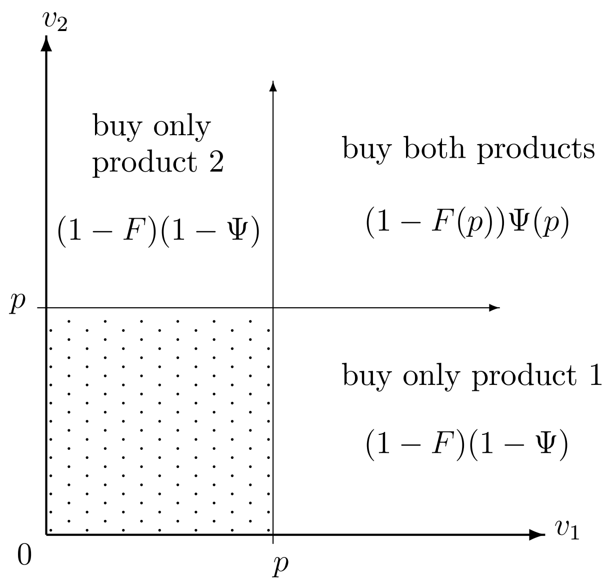

Figure 1: Pattern of demand with additive valuations

3 Bundling Revisited

Consider a market with two products, labeled 1 and 2, where there is a constant marginal cost of supplying product i equal to c_i. Depending on the context, we will consider situations where a monopolist supplies both products, as well as situations where the two products are supplied by separate firms. Each consumer wishes to buy either zero or one unit of each product. A consumer is willing to pay v_i for product $i = 1, 2$ on its own, and to pay v_{12} for the bundle of both products. Thus a consumer's preferences are entirely described by the vector (v_1, v_2, v_{12}), and this vector is distributed across the population of consumers according to some known distribution. Unlike most of the bundling literature, we allow for non-additive preferences so that $v_{12} \neq v_1 + v_2$. We say that a consumer views the two products as (partial) substitutes whenever $v_{12} \leq v_1 + v_2$. Whenever there is free disposal (so that a consumer can discard one item without incurring any cost), we require that $v_{12} \geq \max\{v_1, v_2\}$ for all consumers.

Consumers face three relevant prices: p_1 is the price for consuming product 1 on its own; p_2 is the price for product 2 on its own, and $p_1 + p_2 - \delta$ is the price for consuming the bundle of both products. Thus, δ is the discount for buying both products (which is zero if there is a linear price for each product, or negative if consumers are charged a premium for joint consumption). A consumer will purchase the option which leaves her with the highest net surplus, i.e., she will buy both items whenever

$$v_{12} - [p_1 + p_2 - \delta] \geq \max\{v_1 - p_1, v_2 - p_2, 0\} \ ,$$

she will buy product $i = 1, 2$ on its own whenever

$$v_i - p_i \geq \max\{v_{12} - [p_1 + p_2 - \delta], v_j - p_j, 0\} \ ,$$

and otherwise she will buy nothing.

As functions of the three tariff parameters (p_1, p_2, δ), denote by Q_1 the proportion of potential consumers why buy only product 1, Q_2 the proportion of consumers who buy only product 2, and Q_{12} is the proportion of consumers who buy both products. It will also be useful to define demand functions when no discount is offered, so let $q_i(p_1, p_2) \equiv Q_i(p_1, p_2, 0)$ and $q_{12}(p_1, p_2) \equiv Q_{12}(p_1, p_2, 0)$ be the corresponding demand functions when $\delta = 0$. Products are gross substitutes if demand for one product, $q_i + q_{12}$, is increasing with the other product's price p_j. Products are gross complements if $q_i + q_{12}$ decreases with p_j. In this paper we focus on the case with substitutes, although parallel analysis applies when products are complements. As one would expect, if all consumers view the products as partial substitutes, the products are then gross substitutes:

Lemma 1: *Suppose that $v_{12} \leq v_1 + v_2$ for all consumers. Then demand for product i, $q_i + q_{12}$, weakly increases with p_j.*

Proof: Suppose that $\delta = 0$ so that linear prices are used. A type (v_1, v_2, v_{12}) consumer buys product 1 if and only if

$$\max\{v_{12} - p_1 - p_2, v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ . \tag{4}$$

The left-hand side is the consumer's maximum surplus if she buys product 1 (either as part of a bundle or on its own), while the right-hand side is the consumer's maximum surplus if she does not buy product 1. We claim that difference between the two sides in (4), that is

$$\max\{v_{12} - p_1 - p_2, v_1 - p_1\} - \max\{v_2 - p_2, 0\} \ , \tag{5}$$

is weakly increasing in p_2. (This then implies that the set of consumer types which buys product 1 is increasing, in the set-theoretic sense, in p_2, and so in particular the measure of such consumers is increasing in p_2.) The only way in which expression (5) could strictly decrease with p_2 is if

$$v_{12} - p_1 - p_2 > v_1 - p_1 \text{ and } v_2 - p_2 < 0 \ .$$

However, since products are substitutes we have $v_{12} \leq v_1 + v_2$, which implies that the above pair of inequalities are contradictory. This establishes the result. ■

Importantly, when a bundle discount is offered, this result can be reversed. That is to say, if products are partial substitutes then when a bundle discount is offered, the demand for a product can *decrease* with the stand-alone price of the other product. For instance, if there is a fixed disutility from joint consumption in the sense that $v_{12} = v_1 + v_2 - z$, then if $\delta > z$ the bundle discount outweighs the disutility z and the net result is that the products act like complements, not substitutes, in terms of cross-price elasticities. The observation that a bundle discount can mitigate or overturn the innate substitutability of products will play a major role in the following analysis.

Regardless of whether the underlying products are complements or substitutes, the three discrete purchasing *options* (buy product 1 only, buy product 2 only, or buy both products) are necessarily substitutes, in the sense that cross-price effects are non-negative:

$$\frac{\partial Q_i}{\partial \delta} \leq 0 \ ; \frac{\partial Q_j}{\partial p_i} + \frac{\partial Q_j}{\partial \delta} \geq 0 \ ; \frac{\partial Q_{12}}{\partial p_i} + \frac{\partial Q_{12}}{\partial \delta} \geq 0 \ . \tag{6}$$

(Concerning the second and third inequalities here, note that if price p_i and discount δ rise by the same amount, the price for the bundle is unchanged but the stand-alone price for item i rises.) We also necessarily have symmetry of cross-price effects:

$$\frac{\partial Q_2}{\partial p_1}+\frac{\partial Q_2}{\partial \delta}\equiv\frac{\partial Q_1}{\partial p_2}+\frac{\partial Q_1}{\partial \delta}\ ;\ \frac{\partial Q_{12}}{\partial p_i}+\frac{\partial Q_{12}}{\partial \delta}\equiv-\frac{\partial Q_i}{\partial \delta}\ . \tag{7}$$

Note that the right-hand expression in (7) implies that

$$\left.\frac{\partial(Q_i+Q_{12})}{\partial \delta}\right|_{\delta=0}=-\frac{\partial q_{12}}{\partial p_i}\ , \tag{8}$$

so that the impact of a small bundle discount on the total demand for product i is equal to the impact of a corresponding price cut on the demand for the bundle. To avoid tedious caveats involving corning solutions, suppose that over the relevant range of linear prices there is some two-item demand, so that $q_{12}>0$.

An integrated monopolist: Suppose an integrated monopolist supplies both products. The firm's profit with bundling tariff (p_1,p_2,δ) is

$$\pi=(p_1-c_1)(Q_1+Q_{12})+(p_2-c_2)(Q_2+Q_{12})-\delta Q_{12}\ . \tag{9}$$

Given that the three purchase options are substitutes as in (6), it follows that the most profitable bundling tariff will involve above-cost pricing for each option, so that

$$p_i\geq c_i\ ;\ p_i+p_2-\delta\geq c_1+c_2\ .$$

Consider the firm's incentive to offer a bundling discount $\delta>0$. Starting from any pair of linear prices (p_1,p_2), by differentiating (9) we see that the impact on profit of introducing a small discount $\delta>0$ has the sign

$$\begin{aligned}\left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} &= \left.\left\{(p_1-c_1)\frac{\partial}{\partial \delta}(Q_1+Q_{12})+(p_2-c_2)\frac{\partial}{\partial \delta}(Q_2+Q_{12})-Q_{12}\right\}\right|_{\delta=0}\\ &= -(p_1-c_1)\frac{\partial q_{12}}{\partial p_1}-(p_2-c_2)\frac{\partial q_{12}}{\partial p_2}-q_{12}\ ,\end{aligned} \tag{10}$$

where the second equality follows from (8). Let (p_1^*,p_2^*) be the most profitable linear prices. Therefore,

$$(p_1^*,p_2^*)\text{ maximizes }(p_1-c_1)(q_1+q_{12})+(p_1-c_2)(q_2+q_{12})\ ,$$

which has first-order condition for p_i^* given by

$$q_i+q_{12}+(p_1^*-c_1)\frac{\partial}{\partial p_i}(q_1+q_{12})+(p_2^*-c_2)\frac{\partial}{\partial p_i}(q_2+q_{12})=0\ .$$

If the products are gross substitutes, both price-cost margins are positive, and in particular $(p_2^* - c_2)\frac{\partial}{\partial p_1}(q_2 + q_{12}) \geq 0$ and $(p_1^* - c_1)\frac{\partial}{\partial p_2}(q_1 + q_{12}) \geq 0$. The above first-order condition therefore implies that

$$p_i^* - c_i \geq \frac{q_i + q_{12}}{-\partial(q_i + q_{12})/\partial p_i} \quad for\ i = 1, 2\ . \tag{11}$$

Substituting this pair of inequalities into (10) shows that bundling is profitable whenever condition (12) holds, as summarized in this result:

Proposition 1: *Suppose that products are gross substitutes and that the following elasticity condition holds*

$$\frac{q_1 + q_{12}}{q_{12}} \frac{\partial q_{12}/\partial p_1}{\partial(q_1 + q_{12})/\partial p_1} + \frac{q_2 + q_{12}}{q_{12}} \frac{\partial q_{12}/\partial p_2}{\partial(q_2 + q_{12})/\partial p_2} > 1\ . \tag{12}$$

Then the integrated monopolist has an incentive to offer a discount when its customers buy both products.

Condition (12) is satisfied when demand for the bundle is not "too much" less elastic than the overall demand for each product. A simple sufficient condition for (12) to hold is that each term on the left-hand side is greater than a half, so that a price rise which causes demand for a particular product to fall by 10% causes demand for the bundle to fall by more than 5%.

Two separate sellers: Next, suppose that the two products are supplied by two separate sellers. When firms offer linear prices—i.e., prices which do not depend on whether the consumer also purchases the other product—firm i chooses its price p_i^*, given its rival's price, to maximize $(p_i - c_i)(q_i + q_{12})$. It some circumstances, it may be feasible for a firm to condition its price on whether a consumer also buys the rival product. For instance, a museum could ask a consumer to show her entry ticket to the other museum to claim a discount. The next result shows when a firm has a unilateral incentive to introduce a discount when a customer also buys the other firm's product.

Proposition 2: *Suppose that demand for the bundle is more elastic than demand for firm i's stand-alone product, i.e., that*

$$-\frac{1}{q_{12}} \frac{\partial q_{12}}{\partial p_i} > -\frac{1}{q_i} \frac{\partial q_i}{\partial p_i}\ . \tag{13}$$

Starting from the situation where both firms set the equilibrium linear prices p_1^* and p_2^*, firm i has an incentive to offer a discount on its product to those consumers who buy product j.

Proof: Firm i's equilibrium linear price p_i^*maximizes $(p_i - c_i)(q_i + q_{12})$, so that

$$0 = q_i \left[1 - (p_i^* - c_i)\frac{-\partial q_i/\partial p_i}{q_i}\right] + q_{12}\left[1 - (p_i^* - c_i)\frac{-\partial q_{12}/\partial p_i}{q_{12}}\right] \ . \tag{14}$$

Suppose now that firm i offers a discount $\delta > 0$ from its price p_i^* to those consumers who purchase product j as well. (Those consumers who only buy product i continue to pay p_i^*.) Then firm i's profit is

$$\pi_i = (p_i^* - \delta - c_i)Q_{12} + (p_i^* - c_i)Q_i \ ,$$

and the impact of a small joint purchase discount is governed by the sign of $\frac{d\pi_i}{d\delta}\big|_{\delta=0}$, which from (8) is equal to

$$-q_{12} - (p_i^* - c_i)\frac{\partial q_{12}}{\partial p_i} \ . \tag{15}$$

When (13) holds, the second term $[\cdot]$ in (14) must be strictly negative, i.e., expression (15) is strictly positive. Therefore, offering a small discount for joint purchase will raise the firm's profit. ■

Thus, starting from the scenario in which both firms set their equilibrium linear prices, whenever (13) holds a firm has a unilateral incentive to offer a discount to any consumer who purchases the other item. The reason is straightforward: since demand for the bundle is more elastic than demand for its stand-alone product, a firm wants to offer a lower price to those consumers who also purchase the other product. Thus, discounts for joint purchase can arise even when products are supplied by separate firms and when a firm chooses, and funds, the bundle discount unilaterally.

If condition (13) holds for firm i, then demand for the bundle is more elastic than total demand for that firm's product, and so

$$\frac{q_i + q_{12}}{q_{12}}\frac{\partial q_{12}/\partial p_i}{\partial(q_i + q_{12})/\partial p_i} > 1 \ .$$

Therefore, condition (12) applies (at least for the same pair of linear prices), and so whenever at least one separate seller has an incentive to bundle, we expect that an integrated firm does also (but not necessarily *vice versa*).

In asymmetric cases, it is possible that condition (13) holds for one firm but not for the other. Thus, one firm has an incentive to offer a joint purchase discount when its customers buy the other product, while the other firm does not. On the other hand, it is possible that both firms wish to offer lower prices to its customers when they buy the rival product. The issue then arises as to how this "double" joint purchase discount is implemented. Since in many case a consumer must buy the two items in *order*, both firms cannot simultaneously require proof of purchase from the other seller when they offer their discount. We suppose that each firm i offers price p_i if the consumer only buys its item and it offers the $p_i - \delta_i$ when the consumer buys the rival's product. We suppose that when a consumer buys both products she pays the joint price $p_1 + p_2 - \delta_1 - \delta_2$, so that a consumer enjoys the discount from both firms. This inter-firm bundling scheme could easily be implemented via some kind of joint marketing body or electronic sales platform.[2] Beyond this modest coordination, there is no need for firms to coordinate their tariffs.

Without specifying consumer tastes in more detail, it is hard to derive further results, in particular about equilibrium bundling tariffs (as opposed merely about the incentive to depart from linear pricing). In the next section we specialise the framework in various ways to obtain further insight.

4 Symmetric Analysis and Examples

For maximum transparency of the analysis, suppose now that the two products are symmetric, so that $c_1 = c_2 = c$ and the same measure of consumers have taste vector (v_1, v_2, v_{12}) as have the permuted taste vector (v_2, v_1, v_{12}). As in section 2, let $F(.)$ denote the marginal c.d.f. for either stand-alone valuation v_i, let $X_1(p)$ denote the proportion of consumers who buy a single item when the price for any item is p (so there is no bundle discount) and let $X_2(p)$ denote the proportion of consumers who buy both items when the linear price is p. Formally we have

$$X_1(p) \equiv q_1(p,p) + q_2(p,p) \ ; \ X_2(p) \equiv q_{12}(p,p) \ .$$

[2] For instance, a website could display the total prices for the various options, and firms receive directly the revenue from their stand-alone products as well as their share of the revenue of the bundle. Similarly, TV channels sold via a broadcasting platform could choose the prices for viewing a channel conditional on which other channels are purchased, and then viewers choose the appropriate bundle of channels and pay the stipulated price to each channel.

Then the analysis presented in section 2 shows that an integrated monopolist wishes to introduce a bundle discount whenever demand for a single item is less elastic than the demand for both items, so that

$$-\frac{X_1'(p)}{X_1(p)} < -\frac{X_2'(p)}{X_2(p)} \tag{16}$$

(where primes denote the derivative with respect to p). In the following analysis, suppose that the marginal distribution F has an increasing hazard rate, so that

$$\frac{f(v)}{1-F(v)} \text{ strictly increases with } v\ . \tag{17}$$

4.1 Additive valuations

Consider first the special case where valuations are additive, so that $v_{12} = v_1 + v_2$ for all consumers. Then, as discussed in section 2, the integrated firm wishes to offer a bundle discount whenever Long's condition (3) is satisfied.

The next result describes when separate sellers have a unilateral incentive to offer a joint purchase discount:

Proposition 3: *Suppose that products are symmetric and valuations are additive. Starting from the situation where both firms set the equilibrium linear prices, a firm has an incentive to offer a discount to those consumers who buy the other product whenever*

$$\frac{\Psi(P)}{1-F(P)} \textit{ is strictly decreasing} \tag{18}$$

where Ψ is the function (2).

Proof: From the definition of Ψ, when $p_1 = p_2 = p$ we have $q_{12} = (1-F(p))\Psi(p)$ and $q_i = (1-F(p))(1-\Psi(p))$. By symmetry we have

$$-\frac{\partial q_{12}}{\partial p_i} = -\tfrac{1}{2}\frac{d}{dp}((1-F)\Psi) = \tfrac{1}{2}(f\Psi - (1-F)\Psi')$$

and

$$-\frac{\partial q_i}{\partial p_i} = f - \tfrac{1}{2}(f\Psi - (1-F)\Psi')\ .$$

(The latter expression follows from the observation that $-\partial(q_i+q_{12})/\partial p_i = -d(1-F)/dp_i = f$.) Thus, condition (13) holds whenever

$$\frac{\tfrac{1}{2}(f\Psi - (1-F)\Psi')}{(1-F)\Psi} > \frac{f - \tfrac{1}{2}(f\Psi - (1-F)\Psi')}{(1-F)(1-\Psi)}$$

i.e., when

$$-f\Psi - (1-F)\Psi' > 0$$

which corresponds to expression (18). Thus Proposition 2 implies the result. ■

Clearly, (18) is a stronger condition than (3). In broad terms, condition (18) requires negative correlation between v_1 and v_2. (If valuations v_1 and v_2 are independent, then $\Psi/(1-F) \equiv 1$.) More precisely, let

$$H(p \mid v) \equiv \Pr\{v_2 \leq p \mid v_1 = v\} \tag{19}$$

be the conditional probability that one value is below p given that the other value equals v. Then a sufficient condition for (18) to hold is that $H(p \mid v)$ strictly increases with v, which is strong form of negative correlation.[3]

It is intuitive that negative correlation is associated with the incentive to engage in inter-firm bundling when valuations are additive (see Schmalensee, 1982, for earlier discussion of this point). If firm i knows that a potential consumer has purchased firm j's product, i.e., the consumer has a relatively high value for item j, then negative correlation implies that this is "bad news" for the consumer's likely value for i's product, and this will usually induce the firm to lower its price to this consumer. By contrast, if there is no correlation in the values for the two items, the observation that a consumer has purchased item j gives no reason for firm i to adjust its price.

To illustrate, consider the family of distributions for $(v_1, v_2) \in [0,1]^2$ with joint c.d.f. given by

$$G(v_1, v_2) = v_1 v_2 + \gamma v_1 v_2 (1-v_1)(1-v_2) \ .$$

(This joint distribution is a copula, of the so-called Fairlie-Gumbel-Morgenstern class.) The marginal c.d.f. for each v_i is uniform, so that $F(v_i) = v_i$. The parameter $-1 \leq \gamma \leq +1$ represents the extent of correlation ($\gamma > 0$ implies positive correlation and $\gamma < 0$ implies

[3] Since

$$(1-F(p))\Psi(p) = \int_p^\infty f(v)(1-H(p \mid p))dv \ ,$$

we have

$$\Psi'(p) = \frac{f(p)}{1-F(p)} \left[\Psi(p) - 2(1-H(p \mid p))\right] \ .$$

It follows that (18) holds whenever $1 - H(p \mid p) > \Psi(p)$, and a sufficient condition for this to be true is that $H(p \mid v)$ strictly increases with v.

negative correlation), and we have $\Psi(p) = (1-p)(1+\gamma p^2)$. So Ψ is strictly decreasing in p for all γ in the relevant range, and an integrated monopolist will always wish to introduce a bundling discount. On the other hand, $\Psi/(1-F) = 1+\gamma p^2$ decreases in p only if $\gamma < 0$, and so Proposition 3 implies that separate sellers wish to offer a unilateral bundle discount only when there is negative correlation in tastes for the two products.

4.2 Bundling substitute products by an integrated monopolist

Now suppose that the products are substitutes, so that $v_{12} \leq v_1 + v_2$ for all consumers. For a type (v_1, v_2, v_{12}) consumer, define

$$v_{[1]} = \max\{v_1, v_2\}$$

to be her maximum utility if she buys only one item, and

$$v_{[2]} = v_{12} - \max\{v_1, v_2\}$$

to be her incremental utility from buying two items rather than one. The assumption that products are substitutes implies that

$$v_{[2]} = v_{12} - \max\{v_1, v_2\} \leq v_1 + v_2 - \max\{v_1, v_2\} = \min\{v_1, v_2\} \leq v_{[1]}$$

so that the support of $(v_{[1]}, v_{[2]})$ lies under the 45^0 line, as shown on Figure 2. Note that, by construction, we have $v_{12} = v_{[1]} + v_{[2]}$, so that valuations are "additive" after this change of variables.

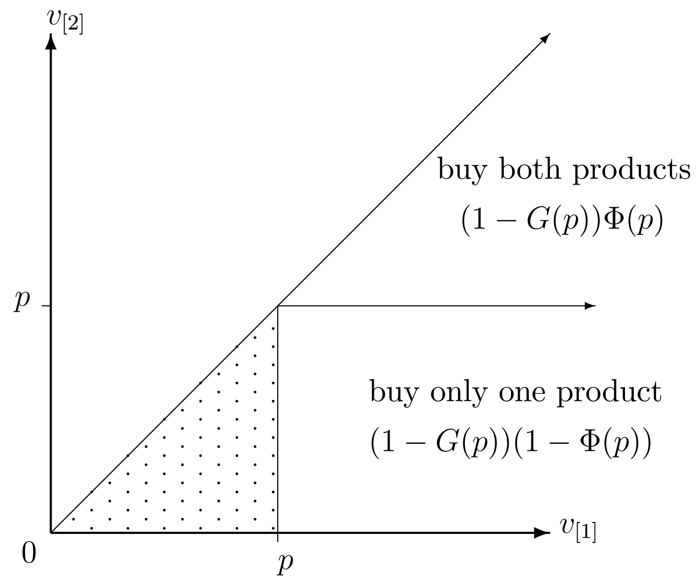

Figure 2: Pattern of demand with substitutes

With a linear price p for either item, a type $(v_{[1]}, v_{[2]})$ consumer will buy both items whenever $v_{[2]} \geq p$, and will buy only one item whenever $v_{[1]} \geq p$ and $v_{[2]} < p$, as depicted on the figure. As in expression (2), define

$$\Phi(p) \equiv \Pr\{v_{[2]} \geq p \mid v_{[1]} \geq p\} \ . \tag{20}$$

When products are partial substitutes rather than independent, this often makes the integrated firm's demand less elastic. If we write $G(P) \equiv \Pr\{v_{[1]} \leq P\}$ for the marginal c.d.f. for $v_{[1]}$, by examining Figure 2, one sees that with linear price p the total number of units demanded is $(1-G(p))(1+\Phi(p))$.[4] To see that an integrated firm offered substitutes will, all else equal, tend to set a higher price than the corresponding firm selling independent products, consider the two polar cases where valuations are additive and where each consumer wants only a single item. In the former case, $v_{[2]} = \min\{v_1, v_2\}$, and Figure 1 shows that

$$\Phi_{ADD}(p) = \Psi(p)/(2 - \Psi(p)) \ , \tag{21}$$

and the firm's most profitable price maximizes $(p-c)(1-G(p))(1+\Psi(p)/(2-\Psi(p)))$. In the latter case, $v_{[2]} = 0$, so

$$\Phi_{SUB}(p) = 0 \ ,$$

and the firm chooses p to maximize $(p - c)(1 - G(p))$. A simple revealed preference argument shows that the latter price is higher than the former whenever Ψ is decreasing. More generally, we expect that more pronounced substitutability between products will usually induce the integrated firm to set a higher linear price.

The following result, which is derived almost immediately, describes when the firm wishes to use a bundle discount:

Proposition 4: *Suppose products are substitutes and Φ in (20) is strictly decreasing. Then an integrated monopolist has an incentive to offer a bundle discount.*

Proof: By writing $X_2(p) = (1-G(p))\Phi(p)$ and $X_1(p) = (1-G(p))(1-\Phi(p))$, as shown on Figure 2, one can verify that condition (16) holds whenever Φ is strictly decreasing. ■

[4] In terms of the marginal c.d.f. $F(\cdot)$ and correlation function $\Psi(\cdot)$ in (2), we have

$$1 - G(p) = (1 - F(p))(2 - \Psi(p)) \ .$$

Proposition 4 generalizes Long's original condition (3) to the case where products are partial substitutes. (When valuations are additive, Φ is given by expression (21), which decreases if and only if (3) holds.) To illustrate, consider an example where the stand-alone values (v_1, v_2) are uniformly distributed on the unit square $[0,1]^2$ and given (v_1, v_2) suppose that the bundle value v_{12} is uniformly distributed on the interval $[\max\{v_1, v_2\}, v_1 + v_2]$. (With free disposal we require that v_{12} not be below $\max\{v_1, v_2\}$, and we require that $v_{12} \leq v_1 + v_2$ if products are substitutes.) Then the density for $v_{[2]}$ given $v_{[1]}$, where $v_{[2]} \leq v_{[1]}$, is

$$\frac{1}{v_{[1]}} \log \frac{v_{[1]}}{v_{[2]}} \ .$$

Therefore, if $P < v_{[1]}$ then

$$\Pr\{v_{[2]} \geq p \mid v_{[1]}\} = 1 - \frac{p}{v_{[1]}} \left(\log v_{[1]} - \log p + 1 \right) \ .$$

Therefore

$$\Phi(p) = 1 + \frac{2p \log p}{1 - p^2} \ , \tag{22}$$

which is indeed a decreasing function, and so the integrated firm will wish to offer a bundle discount.

If $c = 0$, the integrated monopolist's most profitable linear price maximizes $p(X_1 + 2X_2) = p(1 - G(p))(1 + \Phi(p))$, where $G(p) = p^2$. It follows that the optimal linear price is approximately $p \approx 0.540$, which yields industry profit of 0.406. Note that about 70% of potential consumers buy something given this price, although only 4% of consumers buy both items. A laborious calculation shows that the firm's optimal bundling tariff is

$$p \approx 0.527 \ ; \ \delta \approx 0.149$$

which yields slightly higher industry profit 0.415. Notice that, compared to the corresponding example with additive values ($v_{12} \equiv v_1 + v_2$), the bundling discount is far less pronounced.[5]

Note that in this example, bundling acts to reduce *all* prices paid by consumers, unlike the additive case where the stand-alone price rises when bundling is used. Some intuition

[5] When $c_1 = c_1 = 0$, (v_1, v_2) is uniformly distributed on $[0,1]^2$ and $v_{12} \equiv v_1 + v_2$, then one can check that $p = \frac{2}{3}$ and $\delta = \frac{\sqrt{2}}{3} \approx 0.47$.

for this goes as follows. As discussed earlier in this section, a more pronounced substitutability between products usually leads the firm to raise its price (as it does in this example). Proposition 4 demonstrates that an integrated firm very often wishes to introduce a bundle discount. But a bundle discount acts to mitigate, or even overturn, the impact of substitution, since the reduced price for the second product reduces or reverses the reduced incremental utility due to substitution. Thus, the use of a bundle discount acts endogenously to weaken product substitutability, and this in turn can lead the firm to reduce its (stand-alone) price.

4.3 Constant disutility of joint consumption

Consider now the situation in which for all consumers we have

$$v_{12} = v_1 + v_2 - z \tag{23}$$

for some constant $z > 0$. Here, to ensure free disposal we need to assume that the minimum possible realization of either v_i is greater than z. Then with a linear price p for buying any single item, the pattern of demand is as shown on this figure. Note that margin of consumers who are indifferent between buying only product 1 or only product 2 (the upward diagonal on the figure), represents the extent of competition when the two products are supplied by separate sellers.

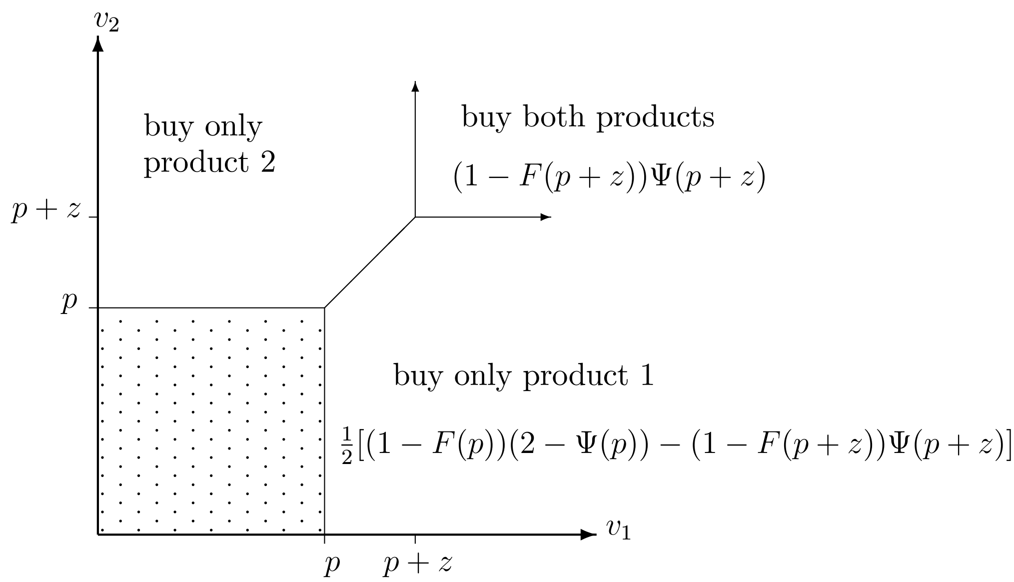

Figure 3: Pattern of demand with constant disutility of joint purchase

Then we have the following condition which ensures that the integrated firm wishes to introduce a bundling discount:

Proposition 5: *Suppose that bundle valuations are given by (23). Then an integrated monopolist has an incentive to offer a bundle discount whenever condition (3) holds.*

Proof: From Figure 3 we see that (20) is given by

$$\Phi(p) = \frac{(1-F(p+z))\Psi(p+z)}{(1-F(p))(2-\Psi(p))} \ .$$

Differentiating shows that Φ is strictly decreasing if and only if

$$\frac{\Psi'(p)}{2-\Psi(p)} + \frac{\Psi'(p+z)}{\Psi(p+z)} < \frac{f(p+z)}{1-F(p+z)} - \frac{f(p)}{1-F(p)} \ .$$

Since F is assumed to have an increasing hazard rate in (17), the right-hand side of the above is non-negative, and if condition (3) holds then the left-hand side is negative. Therefore, Φ is strictly decreasing and Proposition 4 implies the result. ■

We next investigate bundling incentives when the products are supplied by separate firms. Note first that a more pronounced substitutability between products, in the sense that z increases, tends to cause a firm's demand to become more elastic, since the competitive frontier in Figure 3 (the upward-sloping margin between consumers who buy only product 1 and consumers who buy only product 2) lengthens. Thus, we expect that competing firms will then set lower equilibrium prices. (Thus, with separate sellers the typical impact of substitution is opposite to that when an integrated firms supplies both products.)

For simplicity, we focus on the situation where v_1 and v_2 are independently distributed. (From section 4.1, we already know that negative correlation will tend to give an incentive to offer a unilateral bundle discount.) The next result shows that a firm typically does have a unilateral incentive to offer a bundle discount.

Proposition 6: *Suppose that v_1 and v_2 are independently distributed and that the bundle valuations satisfy (23). When the two products are supplied by separate sellers, each seller has an incentive to offer a discount to those consumers who buy the rival product.*

Proof: By examining Figure 3, we see that

$$-\frac{\partial q_{12}}{\partial p_1} = f(p+z)(1-F(p+z))$$

and

$$-\frac{\partial q_1}{\partial p_1} = f(p)F(p) + \int_p^{p+z} (f(v))^2 dv$$

(where these derivatives are evaluated at symmetric prices $p_1 = p_2 = p$). At the symmetric price p we have

$$q_{12} = (1-F(p+z))^2 \ ; \ q_1 = \tfrac{1}{2}\left(1-(F(p))^2-(1-F(p+z))^2\right) \ .$$

We need to show that inequality (13) holds so that Proposition 2 can be applied.

Since F has an increasing hazard rate in (17), we have

$$\begin{aligned}
\int_p^{p+z} (f(v))^2 dv &= \int_p^{p+z} \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\
&\leq \frac{f(p+z)}{1-F(p+z)} \int_p^{p+z} f(v)(1-F(v))dv \\
&= \frac{1}{2}\frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right) \ .
\end{aligned}$$

Therefore, a sufficient condition for (13) to hold is that

$$\frac{f(p+z)}{1-F(p+z)} > \frac{2f(p)F(p) + \frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right)}{1-(F(p))^2-(1-F(p+z))^2}$$

which can be rearranged to give

$$\frac{f(p+z)}{1-F(p+z)} > \frac{f(p)}{1-F(p)} \ .$$

Since F has a strictly increasing hazard rate, the claim is established. ■

Similarly to the discussion in section 4.1, it is economically intuitive that products being substitutes of the form (23) will give an incentive to a firm to offer a discount when its customers have purchased the rival product. If the potential customer has already purchased the other product, this is "bad news" for the firm as the customer's incremental value for its product has been shifted downwards by z, and typically this will give an incentive to offer the customer a lower price. (See Lewbel, 1985, for earlier discussion of this point.)

To illustrate, consider the following example. Suppose that (v_1, v_2) is uniformly distributed on the unit square $[1, 2]^2$, that $z = \frac{1}{2}$ and that $c = 1$.[6] Then an integrated monopolist which sets linear prices will choose the linear price

$$p = 1 + \frac{1}{\sqrt{3}} \approx 1.58 \ ,$$

and this generates profit $\frac{2}{3\sqrt{3}} \approx 0.385$. Since $p + z > 2$, at this price there is no two-item demand at all (see Figure 3). By contrast, the most profitable bundling tariff is

$$p = 1 + \frac{1}{\sqrt{3}} \approx 1.58 \ ; \ \delta = \frac{1}{\sqrt{3}} - \frac{1}{6} \approx 0.41 \ ,$$

which generates profit of about 0.403, and one in nine consumers buy both items. In particular, and similarly to the example presented in section 4.2, the use of bundling means that the firm weakly lowers *all* its prices, thus boosting both consumer surplus and total welfare.[7]

Consider next the case with separate sellers in the same example. Then the equilibrium with linear pricing has price $p = 17/12 \approx 1.417$ and industry profit is about 0.347. Consumer surplus is around 0.274. Less than 1% of consumers buy both items with this linear price. Numerical calculations show that the equilibrium inter-firm bundling tariff is

$$p_1 = p_2 = 1.454 \ ; \ \delta_1 = \delta_2 = 0.102 \ .$$

Thus, the discount if a consumers buys the second product is about 15%. Industry profit is now 0.376, and around 6% of consumers buy both items. However, relative to linear pricing, consumer surplus falls to 0.245. In particular, the use of inter-firm discounts may harm consumers, despite their apparently pro-consumer effect. Intuitively, when firms offer a bundle discount, this reduces the effective degree of substitution between products, which in turn relaxes competition between firms. In particular, and in contrast to the case of an integrated firm, when bundling is used the regular price increases relative to the situation with linear pricing.

[6] Note that this example gives rise to a linear demand system when linear prices are used, and when prices are such that there is some two-item demand and some consumers who buy nothing, Figure 3 shows that

$$\text{demand for product } i = k + \tfrac{1}{2}p_j - p_i \ .$$

[7] Note that in this example, since there is no two-item demand with linear pricing, the firm has no *local* incentive to introduce a bundle discount, although it does have a global incentive to do so.

4.4 Time-constrained consumers

A natural reason why products might be substitutes is that some buyers are only able to consume a restricted set of products, e.g., due to time constraints. For instance, a tourist may have the time only to visit a single museum in a city.

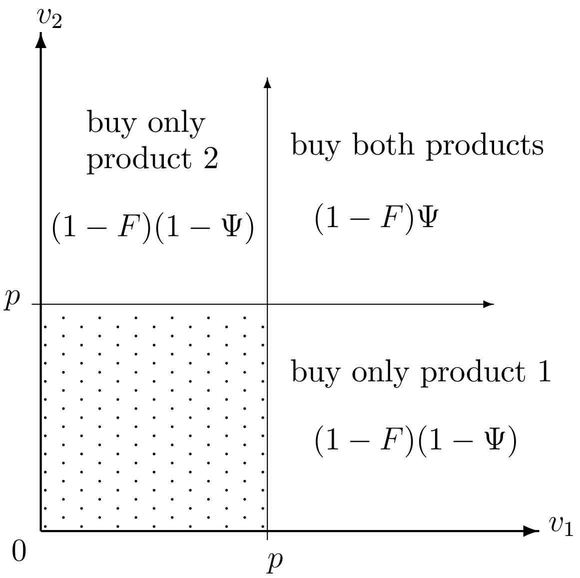

Figure 4A: Additive valuations

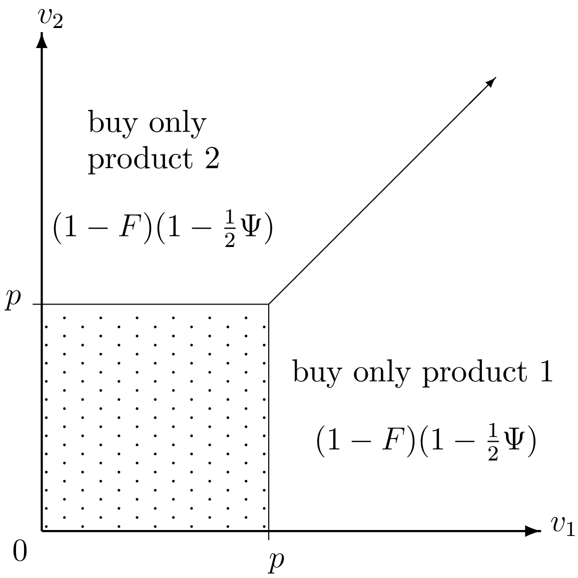

Figure 4B: Time-constrained consumers

Suppose that an exogenous fraction λ of consumers have valuation v_i for stand-alone product $i = 1, 2$ and valuation $v_{12} = v_1 + v_2$ for the bundle, while the remaining consumers can only buy one item (and have valuation v_i for item i). For simplicity, suppose that the distribution for (v_1, v_2) is the same for the two groups of consumers. Let the marginal c.d.f. for each v_i be $F(v)$, and let $\Psi(\cdot)$ be as defined in (2). (See Figure 4 for an illustration.) The central feature of this scenario is that the time-constrained consumers have zero incremental value for the second item (so for them $v_{[2]} = 0$). It is then straightforward to show that

$$\Phi(p) = \lambda \frac{\Psi(p)}{2 - \Psi(p)} \ ,$$

so that Φ is decreasing if and only if Ψ is decreasing. Proposition 4 therefore has the corollary:

Proposition 7: *When some consumers are time-constrained, an integrated firm has an incentive to offer a bundle discount whenever (3) holds, i.e., under the same conditions as when no consumers were time-constrained.*

Finally, consider the situation with separate sellers:

Proposition 8: *Suppose that v_1 and v_2 are independently distributed and that some consumers are time-constrained. When the two products are supplied by separate sellers, a seller has no incentive to offer a discount to those consumers who buy the rival product. (They would, if feasible, like to charge their customers a higher price when a customer buys the rival product.)*

Proof: By examining Figure 4, we see that

$$-\frac{\partial q_{12}}{\partial p_1} = \lambda f(1-F) \ ; \ q_{12} = \lambda(1-F)^2$$

and

$$-\frac{\partial q_1}{\partial p_1} = fF + (1-\lambda)\int_p^\infty (f(v))^2 dv \ ; \ q_1 = \lambda F(1-F) + \tfrac{1}{2}(1-\lambda)(1-F^2)$$

(where these expressions are evaluated at symmetric prices $p_1 = p_2 = p$ and the dependence of f and F on p is suppressed). We need to show that inequality (13) is reversed.

Since F has an increasing hazard rate in (17), we have

$$\begin{aligned}\int_p^\infty (f(v))^2 dv &= \int_p^\infty \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\ &> \frac{f}{1-F}\int_p^\infty f(v)(1-F(v))dv \\ &= \tfrac{1}{2}\frac{f}{1-F}(1-F)^2 \\ &= \tfrac{1}{2}f(1-F) \ .\end{aligned}$$

Thus (13) is reversed whenever

$$\frac{f}{1-F} < \frac{2fF + (1-\lambda)f(1-F)}{2\lambda F(1-F) + (1-\lambda)(1-F^2)}$$

which some rearranging shows to be always the case provided $\lambda < 1$, which establishes the result. ■

This implies that, starting from the situation in which firms set their equilibrium linear price, if feasible a firm would wish to charge a *higher* price to its customers who also buy the rival's product. In this framework, the observation that a consumer wishes to buy both items implies she is in the "non-competitive" group of consumers, and a firm would like to exploit its monopoly position over those consumers if possible. Comparing this result with Proposition 6 shows that the precise form in which products are substitutes is important for a firm's incentive to offer inter-firm bundling discounts.

5 Partial Coordination Between Sellers

The analysis to this point has considered the two extreme cases where (a) there is no tariff coordination between separate sellers, and (b) where there is complete tariff coordination between sellers. (The integrated-firm analysis above describes the outcome when two sellers coordinate their pricing to maximize industry profit.). The problem with complete coordination is that any competition between the rivals is eliminated. It would be desirable, is feasible, to obtain the efficiency gains which may accrue to bundling without permitting the firms to collude over their regular prices. For instance, firms might agree on a bundle discount, but then compete in the usual way by choosing their stand-alone prices independently.

To consider this situation in more detail, suppose that two separate symmetric firms supply the two products. The firms interact in two stages in a similar manner to the procedure in Gans and King (2006). First, the two firms agree on a bundle discount δ, say, which they agree to fund equally. That is to say, if firm $i = 1, 2$ chooses stand-alone price p_i, the consumers pays this price if she buys only that firm's product (and the firm receives that revenue), but if she buys both products she pays $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$. After δ is chosen, firms choose their stand-alone prices unilaterally. Far-sighted firms will choose δ after taking into account how this discount will affect their competitive interaction in the second stage. Since separate firms tend to set lower prices when products are more substitutable, and since a bundle discount mitigates or overturns a consumer's view of the products as substitutes, it will often be the case that an agreed bundle discount δ will induce firms to set higher (stand-alone) prices. To the extent this is so, a joint-pricing scheme of this form could act as an instrument of collusion.

Consider first the case in which valuations are additive. Then for an agreed inter-firm discount δ, the pattern of demand for the two firms is as illustrated in Figure 5. The use of the discount δ will often make a firm's demand less elastic as a function of its own price.

For instance, if (v_1, v_2) is uniformly distributed, then $-\frac{\partial(Q_1+Q_{12})}{\partial p_1}$ does not depend on δ (or is made strictly smaller if δ is so large that no consumer buys only product 2), while the size of the demand $Q_1 + Q_{12}$ is increased. Thus, the elasticity of demand falls. Since a firm's demand is made less elastic when firms agree to an inter-firm discount, the scheme will often lead firms to choose higher regular prices.

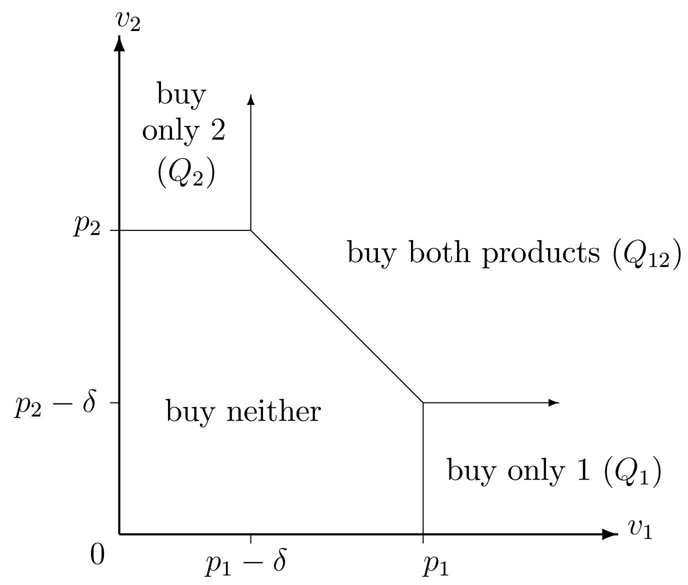

Figure 5: Pattern of demand with additive values and bundling discount $\delta > 0$

The following result shows that this joint pricing scheme leads to higher profit, and describes when the scheme also increases total welfare:

Proposition 9: *Suppose that products are symmetric and valuations are additive. For given $\delta > 0$ consider the following joint pricing scheme: firms are free to set their own stand-alone prices, and if firm i sets the stand-alone price p_i then the price for buying both products is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold. If condition (3) holds, for sufficiently small $\delta > 0$ this inter-firm bundling scheme increases each firm's profit, relative to the situation where the products are marketed independently. If in addition the function $H(p, v)$ in (19) is weakly increasing in v, the scheme also increases total welfare.*

Proof: Firm i's profit under the proposed joint-pricing scheme is

$$(p_i - c)(Q_i + Q_{12}) - \tfrac{1}{2}\delta Q_{12} \ . \tag{24}$$

The impact of introducing a small $\delta > 0$ on firm i's equilibrium profit is therefore governed by the sign of

$$\frac{d}{d\delta}\left\{(p_i - c_i)(Q_i + Q_{12}) - \tfrac{1}{2}\delta Q_{12}\right\}\Big|_{\delta=0}$$

$$= \frac{dp_i}{d\delta}\frac{\partial}{\partial p_i}[(p_i - c_i)(Q_i + Q_{12})]\Bigg|_{\delta=0, p_i=p_i^*} + \frac{dp_j}{d\delta}\frac{\partial}{\partial p_j}[(p_i - c_i)(Q_i + Q_{12})]\Bigg|_{\delta=0, p_i=p_i^*} \tag{25}$$

$$- \tfrac{1}{2}Q_{12}\big|_{\delta=0} + (p_i^* - c_i)\frac{\partial}{\partial\delta}(Q_i + Q_{12})\Bigg|_{\delta=0}$$

$$= -\tfrac{1}{2}q_{12} - (p^* - c)\frac{\partial q_{12}}{\partial p_i} \tag{26}$$

(where this final expression is evaluated at optimal linear price p^*). Here, the terms in line (25) vanish, the first because p^* is the optimal price for firm i when firms choose linear prices (i.e., p_i maximizes $(p_i - c_i)(q_i + q_{12})$), and the second because changing the other firm's price has no impact on a firm's demand when there is no bundling discount (i.e., $q_i + q_{12}$ does not depend on p_j when values are additive). The final expression follows from (8). Following by-now familiar arguments, the term (26) is strictly positive whenever (3) holds.

Consider the impact of the joint pricing scheme on total welfare. To calculate this we need to understand how the introduction of δ affects equilibrium prices p_i. Firm i's profit is given by (24) and so the first-order condition for p_i to be optimal given δ (and p_j) is

$$Q_i + Q_{12} + (p - c)\frac{\partial(Q_i + Q_{12})}{\partial p_i} - \frac{1}{2}\delta\frac{\partial Q_{12}}{\partial p_i} = 0\ . \tag{27}$$

This expression then determines equilibrium price $p(\delta)$ as a function of the discount δ. Totally differentiating (27) with respect to δ yields

$$0 = \frac{\partial(Q_i + Q_{12})}{\partial\delta} + 2p'\frac{\partial(Q_i + Q_{12})}{\partial p_i} + p'\frac{\partial(Q_i + Q_{12})}{\partial p_j}$$

$$+(p - c)\left[\frac{\partial^2(Q_i + Q_{12})}{\partial p_i\partial\delta} + p'\frac{\partial^2(Q_i + Q_{12})}{\partial p_i^2} + p'\frac{\partial^2(Q_i + Q_{12})}{\partial p_i\partial p_j}\right] - \frac{1}{2}\frac{\partial Q_{12}}{\partial p_i}\ ,$$

where $p' = \frac{d}{d\delta}p(\delta)$ stands for how the equilibrium price depends on the agreed discount δ. When $\delta = 0$ this simplifies to

$$0 = -\frac{3}{2}\frac{\partial q_{12}}{\partial p_i} - 2fp' + (p - c)\left[-\frac{\partial^2 q_{12}}{\partial p_i^2} - p'f'\right]\ . \tag{28}$$

Note that

$$-\frac{\partial q_{12}}{\partial p_1} = f(p_1)(1 - H(p_2 \mid p_1))$$

and so

$$
\begin{aligned}
-\left.\frac{\partial^2 q_{12}}{\partial p_1^2}\right|_{p_1=p_2=p} &= f'(p)(1-H(p\mid p)) - f(p)\frac{\partial}{\partial p_1}H(p_2\mid p_1) \\
&\leq f'(p)(1-H(p\mid p)) \\
&= -\frac{f'(p)}{f(p)}\frac{\partial q_{12}}{\partial p_1}\ , \qquad (29)
\end{aligned}
$$

where the inequality follows from the assumption that $H(p \mid v)$ weakly increases in v. Thus, expression (28) implies

$$
\begin{aligned}
[2f+(p-c)f']p' &= -\frac{3}{2}\frac{\partial q_{12}}{\partial p_i} - (p-c)\frac{\partial^2 q_{12}}{\partial p_i^2} \\
&\leq -\frac{\partial q_{12}}{\partial p_i}\left[\frac{3}{2}+\frac{f'}{f}(p-c)\right] \\
&\leq -\frac{\partial q_{12}}{\partial p_i}\left[2+\frac{f'}{f}(p-c)\right] \\
&= -\frac{1}{f}\frac{\partial q_{12}}{\partial p_i}\,[2f+f'(p-c)]\ .
\end{aligned}
$$

Here, the first inequality follows from (29), and the second follows from the fact that $\frac{\partial q_{12}}{\partial p_i}$ is negative. Since the term $[2f+f'(p-c)]$ is strictly positive due to the second-order condition for p to the equilibrium price when $\delta=0$, we deduce that

$$
fp' \leq -\frac{\partial q_{12}}{\partial p_i}\ . \qquad (30)
$$

By inspecting Figure 5, one can see that the impact of a small discount δ on total welfare is equal to

$$
W' = 2f(p)(p-c)\left\{(1-H(p\mid p))(1-p') - H(p\mid p)p'\right\}\ .
$$

(Here, the first term represents the welfare gain when more single-item consumers buy two items, as the incremental cost of the second item falls to $p(\delta)-\delta$, while the second term represents the welfare loss when some marginal single-item consumers decide to buy nothing due to the price rising to $p(\delta)$.) This welfare change has the sign of

$$
W' \overset{\text{sgn}}{=} f\left\{1-H-p'\right\} = -\frac{\partial q_{12}}{\partial p_i} - fp' \geq 0\ ,
$$

where the inequality follows from (30). Thus, when $H(p \mid v)$ increases with v, the joint pricing scheme will increase total welfare when δ is small. ■

The reason that a small agreed inter-firm discount will boost profits is relatively intuitive. A small $\delta > 0$ will have some effect the firms' choice of stand-alone price, but this has no first-order impact on a firm's profit. (A small change in the firm's own price does not significantly affect the firm's profit, since the original price was at the optimal level. And a small change in the other firm's price does not significantly affect the firm's profit when the bundle discount small, as can be seen from Figure 5.) The first-order impact on industry profit is that, for a *fixed* stand-alone price p, the introduction of a bundle discount boosts profit whenever expression (3) is satisfied. The impact on total welfare is more complex, as the impact of the discount on equilibrium prices needs to be considered. While a bundle discount tends to induce firms to raise their stand-alone prices, when values are independently distributed or negatively correlated, the impact of the price rise is less marked than the efficiency benefits of the bundle discount, and total welfare rises when the scheme is used. (In practice, this will also be true when there is a "moderate" degree of positive correlation in values, as there is some slack in the argument in the proof of Proposition 9.)

To illustrate, consider the example where (v_1, v_2) is uniformly distributed on the unit square $[0,1]^2$ and $c = 0$. Then using Figure 5, one can show that each firm's equilibrium stand-alone price as a function of the agreed discount is

$$p(\delta) = \frac{3\delta + 2\delta^2 + 2}{3\delta + 4} \ ,$$

which is indeed increasing in δ. For small δ, we have shown that this scheme benefits the firms and efficiency.[8] However, in this example the scheme harms aggregate consumer surplus.[9]

While the operation of this joint pricing scheme appears to be relatively benign when values are additive, this can easily be reversed when firms offer substitutable products. Consumers benefit, and total welfare rises, when firms are forced to set low prices due to products being substitutes. However, an agreed inter-firm discount can reduce the effective substitutability of products, and thus relax competition between suppliers. While

[8] One can check that the most profitable choice of δ for the firms is $\delta \approx 0.38$.

[9] When $\delta = 0$, we have $p' = \frac{3}{8}$. Therefore, when δ is small that half of the consumer population who only buy one item experience a price rise of $\frac{3}{8}\delta$, while that quarter of consumers who buy both items experience a price fall of $\frac{1}{4}\delta$. Thus, the net impact on consumers is a loss of $\frac{1}{8}\delta$.

this effect can be demonstrated much more generally, for maximum clarify consider the following simple example:

Example: There are two museums in a city, and the marginal cost of a museum visit is zero. All visitors have identical tastes, and the two museums are homogenous in the sense that if a visitor goes to just one museum, she does not mind which one it is. A visitor values visiting any single museum at V_1 and gains incremental utility $V_2 < V_1$ from visiting the second museum. Because of the declining marginal value of visits, the two museums compete to some extent. If each museum sets an independent entry charge, one can check that the equilibrium entry fee is the incremental value of a second visit, V_2. The result is that consumers visit both museums, and obtain strictly positive surplus $V_1 - V_2$. Suppose the two museums are free to choose their own entry charge but agree in advance to offer a discount δ on the sum of stand-alone prices if a consumer visits both attractions, and they fund this discount equally. (That is to say, if museum i chooses the entry fee p_i, the charge for visiting both museums is $p_1 + p_2 - \delta$ and museum i receives revenue $p_i - \frac{1}{2}\delta$ when a consumer visits both attractions.) One can check that the equilibrium stand-alone price with discount $\delta \leq V_1 - V_2$ is $p = V_2 + \delta$, with the result that consumers visit both museums and pay the joint price $2V_2 + \delta$.[10] In particular, by choosing $\delta = V_1 - V_2$ firms can induce the fully collusive outcome. Thus, an apparently pro-consumer policy of offering a discount for joint purchase can act as a device to sustain collusion. Note that this device works even when firms supply perfect substitutes (i.e., $V_2 = 0$).

6 Conclusions

[To be written...]

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[10] If both firms offer the price $V_2 + \delta$, the bundle price is $2V_2 + \delta$, and each consumer is (just) willing to buy the bundle rather than a single item. If a firm deviates and sets a lower price, no consumer switches to buying *only* that firm's product, and so only bundles are sold and the firm simply obtains less revenue from each bundle. On the other hand, if the firm deviates to a higher price, all consumers will switch to buying *only* the other firm's product.

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