

# Corporate Control and Executive Selection\*

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## Abstract

We present a model in which the owner of the firm enjoys a private benefit from developing a personal relationship with the executives. This may lead the owner to retain a senior executive in office even though a more productive replacement is available. The model shows that the private returns of the employment relationship distort executive selection, reducing the executives' average ability and the firm productivity. We estimate the structural parameters of the model using a panel of Italian firms with information on the nature of the controlling shareholder, matched with individual records of their executives. These estimates are used to quantify the relevance of private returns and the related productivity gap across firms characterized by four different types of ownership: government, family, conglomerate and foreign. We find that private returns are large in family and government controlled firms, while smaller with other ownership types. The resulting distortion in executive selection can account for TFP differentials between control types of about 10%. The structural estimates are fully consistent with a set of model-based OLS regressions, even though the sample moments used by the two approaches are different.

Key Words: corporate governance, private returns, TFP.

JEL classification numbers: D2, G32, L2

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# 1 Introduction

This paper studies executive selection in firms with concentrated ownership, a control structure that the recent corporate finance literature has shown to be very diffused around the world (La-Porta, Lopez-De-Silanes, and Shleifer, 1999; Faccio and Lang, 2002). Unlike public corporations, where the separation between ownership and control naturally puts agency issues at center stage,<sup>1</sup> our hypothesis is that in firms with concentrated ownership the controlling shareholder may pursue some private returns, such as electoral goals in a firm controlled by politicians or family prestige in a firm controlled by an individual.<sup>2</sup> We present a model to analyze how this mechanism distorts executive selection and reduces firm productivity. Two structural estimates of the model, based on different sample moments, indicate that this mechanism is quantitatively important.

We focus on a specific form of *owner's* private benefits and study how it affects the process of executive selection. We assume that owners might derive utility not only from profits but also from employing executives with whom they have developed personal ties. Personal ties and repeated interaction can facilitate the delivery of non monetary payoffs that are typically not verifiable in court and therefore cannot be part of the employment contract. For example, the owner of a family business might enjoy a compliant entourage and/or a group of executives who pursue family prestige, possibly at the expense of the value of the firm.<sup>3</sup> A politician (the “owner” of a government controlled firm) might want executives who serve his political interests, resulting in, e.g., hiring workers in his constituency. Finally, diverting resources at

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<sup>1</sup>See [Tirole \(2006\)](#) for a comprehensive treatment of the agency approach to corporate finance. In [Section 2](#) we provide a brief review of the related literature.

<sup>2</sup>Consistent with the hypothesis that control commands a premium, [Dyck and Zingales \(2004\)](#) provide cross-country evidence that controlling blocks are sold on average at a 14% premium, up to 65% in certain countries—see below for more details. The importance of private benefits of control is also stressed by [Moskowitz and Vissing-Jørgensen \(2002\)](#), who show that in the US average returns of privately held firms are dominated by the market portfolio. They conclude that owners of private firms must be obtaining some form of non-monetary return.

<sup>3</sup>Becker was the first to stress the importance of non-pure consumption components of preferences for individual decision making. In the introduction to the book collecting his contributions on this topic, he states that “Men and women want respect, recognition, prestige, acceptance, and power from their family, friends, peers, and others” ([Becker, 1998](#), p. 12).

the expenses of minority shareholders or plain wrongdoing requires obliging executives, as some recent corporate scandals have shown. If owners value personal ties, they might do so at the expenses of ability, thus distorting the process of executive selection with respect to a situation where only the value of the firm matters. There is indeed evidence, reviewed below, that social networks and personal ties can be detrimental for firm performance (Landier, Sraer, and Thesmar, 2006; Kramarz and Thesmar, 2006).

We consider a simple partial-equilibrium infinite horizon economy in which a firm owner chooses the firm’s executives. Following Lucas (1978) we assume that average managerial ability determines the firm TFP.<sup>4</sup> Executives are characterized by two random variables: their ability (productivity)  $x$  and their relationship value  $r$ , whose distribution is known to the firm owner. Both variables are specific to the executive-firm match, and the owner only learns their realization after an in-office trial period for the executive. Upon learning  $x$  and  $r$  the owner decides whether to give tenure to the executive, who in this case turns “senior”, or to replace him with a “junior” one. We assume that relationship-building takes time, so that only senior executives may deliver the private returns from the personal relationship. In particular, we assume the expected value of  $r$  is zero for a junior executive, while  $\mathbb{E}(r) = qR$  for a senior, where  $q$  denotes the ex-ante probability that the senior executive delivers a valuable relationship and  $R$  is the value of the relationship. The key decision for the firm owner is whether to retain the executive in the firm after learning the value of  $s = x + r$ , or to replace the executive with a junior one. Once tenured, executives in office die with an exogenous probability  $\rho$ . A standard model where the owner maximizes the value of the firm is obtained as a special case assuming that  $R = 0$ .

For a given distribution of ability  $G(x)$  and hazard  $\rho$  the model provides a mapping between the fundamental parameters  $q, R$  and two firm-level observations: the average productivity  $X$  and the share of senior executives  $\phi$ . Our objective is to use this mapping to

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<sup>4</sup>Literally, in Lucas (1978), TFP depends on the *entrepreneurial*, rather than managerial, ability. But the same reasoning can be applied to executives, especially for medium to large size firms like those in our sample, run by a pool of executives rather than by a single entrepreneur.

quantify the effects on  $X, \phi$  of different control types, modeled as different values of  $R$ . A natural interpretation of  $R$  is that it provides a measure of the importance that different owner types assign to private-returns, measured in units of the firm-level productivity. We use the distribution of ability  $G(x)$  to capture systematic differences between industrial sectors, e.g., differences in average TFP across sectors of different technological intensity. We explain the residual variation in  $X, \phi$  by variation in the nature of the control type, i.e.,  $R$ , and one value of  $q$  common to all control-types. In our model the heterogeneity in the value of personal ties  $R$  gives rise to persistent cross-sectional differences in firms' productivity, in accordance with the findings of a vast empirical literature (see [Syverson, 2010](#), for a recent survey). We show that given the distribution of abilities  $G(x)$  there is a one-to-one mapping between  $X, \phi$  and the two structural parameters  $q, R$ . We invert this mapping to estimate the private benefits of control using a large sample of matched firm-executives data across four different types of control: family, government, conglomerate and foreign.

The structural estimates are based on a sample of Italian manufacturing firms for which we have detailed information on the firms' characteristics, including the complete work history of their executives. We construct TFP using the [Olley and Pakes \(1996\)](#) procedure and define senior executives those who have been with the firm for at least five years. The data also classify the controlling shareholder into the four broad control types listed above (individual/family, the government, a conglomerate, or a foreign institution). We allow the importance of private returns  $R$  to vary across control types, without imposing any specific pattern in the estimation. The estimates of the model's fundamental parameters  $q, R$  and the ability distribution  $G(x)$  are used to quantify the relevance of private returns and the related efficiency losses for each of the four ownership types. We find that the executive selection is distorted with respect to the benchmark of no private values ( $R = 0$ ) in all control types. The distortion is smallest for conglomerate and foreign controlled firms. The importance of private benefits accounts for a decrease in average managerial ability (i.e., firm productivity) of around 6% in family firms and of 10% in government firms as compared to

conglomerate and foreign controlled firms. According to our estimates, this happens because controlling shareholders of family and government firms select executives almost exclusively on the basis of personal ties: they tend to keep all the executives with whom they developed a relationship, independent of ability, and fire all the others. This mechanism inhibits the selection effect of managerial ability.

Another implication of the model is that the correlation across firms between productivity and the share of senior executives for a given owner type measures the strength of the selection effect on managerial ability. We prove that this correlation can be estimated by an OLS regression, which offers an alternative way to identify a subset of the model parameters. This estimation strategy uses a different source of data variability with respect to the structural estimates: while the latter exploits the variation in average sample moments across control types, the OLS regression is based on the correlation within a given control type between productivity and the share of senior executives at the firm level. As such, it controls for potential unobserved factors affecting the average share of senior executives and average productivity for given control type. Remarkably, the two sets of estimates give very similar results about the effect of private returns on productivity. Moreover, within an OLS framework it is easy to check that the results are robust to a series of modifications, such as changing the length of time used to classify “senior” executives, conditioning on other factors affecting the firm productivity and using alternative measures of productivity or profit-based performance indicators.

The paper is organized as follows. The next section discusses the connection with the literature, in particular with [Bandiera et al. \(2009\)](#) and [Taylor \(2010\)](#), who study a related problem. [Section 3](#) develops a model to study how the presence of private returns affects the selection of executives and average productivity within a firm. In [Section 4](#) we study the mapping between the model and the data, discussing the identification of the structural model parameters. [Section 5](#) describes the matched employer-employee data and the classification of the various types of corporate control. [Section 6](#) presents the structural estimates of

the model parameters and uses them to quantify the “costs” of private benefits in terms of foregone productivity. [Section 7](#) presents an alternative test of the model hypothesis based on a model-based regression analysis. [Section 8](#) concludes.

## 2 Related literature

The idea that private benefits play a central role in shaping firms’ performance is central to the recent corporate governance literature ([La Porta et al., 2000](#)). [Dyck and Zingales \(2004\)](#) empirically estimate the value of private benefits of controls using the difference between the price per share of a transaction involving a controlling block and the price on the stock market before that transaction. They find large values of private benefits of control. In particular, at 37% the value for Italy is the second highest in a sample of 39 countries. Compared to this literature, we focus on a very specific channel through which the private benefits arise: the relationship between the owner and her executives. Moreover, we use the model’s predictions on productivity and executive seniority distribution to estimate the value of such a relationship, rather than referring to stock market data.

In our model, inefficient selection derives from owners’ valuing a personal relationship with the management. The fact that personal ties between firms’ high-ranked stakeholders (large shareholders, board members, top managers) is detrimental for firm performance finds support in recent literature. [Landier, Sraer, and Thesmar \(2006\)](#) study the effects of independent top-ranking executives, defined as those that joined the firm before the current CEO was appointed, on firms’ profitability and returns from takeovers. They assume that top-ranking executives hired after the current CEO was appointed are more likely to implement her decisions in a non-critical way. They find that, for a panel of US listed corporations, the share of independent top executives is positively related to all indicators of firms’ performance. For France, [Kramarz and Thesmar \(2006\)](#) study the effects of social networks on the composition of firms’ board of directors and performance. They find that networks, defined in terms of

school of graduation, influence the board composition; moreover, firms with a higher share of directors from the same network have lower performance. [Bandiera, Barankay, and Rasul \(2008\)](#) study the effects of social connections among managers and workers on performance. Using a field experiment, they show that managers favor workers they are socially connected to, possibly at the expense of the firm's performance.

Our work also relates to the vast body of work that documents that firms of very different productivity levels coexist even within narrowly defined markets (see, e.g., [Bartelsmann and Doms, 2000](#); [Syverson, 2010](#)). As in [Lucas \(1978\)](#), in our model dispersion in firm productivity derives from the underlying dispersion in managerial ability, subject to a cutoff level dictated by the selection effect.<sup>5</sup> Differently from Lucas, in our model owners are willing to accept a low monetary return on their investment because they derive other types of returns, which weakens the selection effect and increases the cross sectional dispersion in firms productivity. Empirically, we find this to be more relevant for family firms, in line with a growing literature on empirical work practices and performance in family firms (see, for example, [Moskowitz and Vissing-Jørgensen, 2002](#); [Bloom and Van Reenen, 2007](#); [Bandiera et al., 2009](#); [Michelacci and Schivardi, 2010](#)).

In terms of managerial turnover, [Volpin \(2002\)](#) studies top executive turnover in Italian listed firms. Consistent with our findings, family controlled firms tend to have lower turnover rates than foreign controlled firms.<sup>6</sup> Executive compensation, promotion policies and turnover are subject to a growing and heterogeneous body of model-based empirical analysis, using, among others, assignment ([Gabaix and Landier, 2008](#); [Terviö, 2008](#)) or moral hazard models ([Gayle, Golan, and Miller, 2009](#)). Compared to this literature, we do not explicitly formalize the market for executives, but focus on the owner's decision to confirm or replace incumbent executives.

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<sup>5</sup>Heterogeneity in an underlying unobservable firm characteristic has become the standard way to model productivity dispersion at the firm level both in IO ([Jovanovic, 1982](#); [Hopenhayn, 1992](#); [Ericson and Pakes, 1995](#)) and in trade ([Bernard et al., 2003](#); [Melitz, 2003](#)).

<sup>6</sup>He also finds that the sensitivity of turnover to performance, an indicator of the quality of the governance, is not significantly different in the two groups. However, his sample has a small number of foreign controlled firms and in fact his main analysis is focussed on family controlled firms only.

Two recent papers are closely related to ours. [Taylor \(2010\)](#) builds and estimates a structural model of CEO turnover with learning about managerial ability and costly turnover. He finds that only very high turnover costs can rationalize the low turnover rate observed in the data. He interprets this result in terms of CEO entrenchment and poor governance. Compared to his paper, we propose a different, possibly complementary, reason for inefficient turnover: owners trade off efficiency for loyalty. We also use data on all top executives, rather than CEOs alone, and exploit different ownership structures to estimate our model parameters. [Bandiera et al. \(2009\)](#) analyze the role of incentive schemes in family and non family firms, assuming that family firms pursue private benefits of control. They show that family firms rely less on performance-based compensation schemes and attract more risk averse and less able managers. The model predictions are supported by reduced-form regressions. We see this paper and ours as complementary. In fact, [Bandiera et al. \(2009\)](#) focus on the optimal compensation scheme, an issue that is ignored in our model. On the other hand, we introduce learning about managerial ability in a dynamic setting, so that we can study turnover and seniority composition. Moreover, we provide direct structural estimates of the importance of private benefits by control type, supplying supporting evidence for a central assumption of the model of [Bandiera et al. \(2009\)](#).

### 3 A model of executive tenure and firm productivity

We model the decision problem of a firm owner in charge of selecting the executives who run the firm. Our aim is to use the model to organize the empirical analysis. Given that we will structurally estimate the model parameters, we keep it as simple as possible. In particular, we focus on the owner's selection problem and completely put aside the market for executives.<sup>7</sup>

A firm employs  $n$  executives, depending on its size (not modeled here). Each executive is

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<sup>7</sup>Our conjecture, to be verified in future work, is that the effects we identify would also hold both in a competitive market for executives and in a search framework. In fact, as long as the private benefits create a surplus, they should affect executive selection in the same way as in our framework, independently from the surplus splitting rule.



characterized by an ability level  $x_i$ . As in Lucas (1978), we assume that ability is a shifter of the production function,  $\bar{x}F(K, L)$ , where  $\bar{x} = \frac{1}{n} \sum_i x_i$  is the average managerial ability,  $K$  the capital stock and  $L$  labor. This assumption has two consequences. First, it implies that we will be able to measure average managerial ability by firm level TFP, which we will use in the empirical section. Second, the fact that overall TFP is additive in individual ability implies that we can study the problem of the owner for each single executive in isolation from the others, as we exclude spillovers in ability among them. This assumption simplifies the analysis in the model that we present next.

### 3.1 The model

The problem describes executive selection by the firm owner (the principal henceforth). The executives are hired at the junior level and become senior - and eligible for tenure - after one period. We think of this period as the time during which executive's quality is learned by the principal. An executive's quality is characterized by two independent exogenous variables: his productivity  $x$ , a non-negative random variable with continuous and differentiable CDF  $G(x)$  with  $G(0) = 0$  and expected value  $\mu = \int_0^\infty x dG(x)$ , and his relationship value  $r$ , a non-negative random variable that is identically zero for a junior executive and equals zero with probability  $1 - q$  or  $R$  with probability  $q$  for a senior executive. The personal relationship is valuable because it facilitates the delivery of non-monetary payoffs, that cannot be explicitly included in an employment contract. For example, a politician might value executives who serve his political interests in government controlled firms, by hiring workers in his constituency. The owner of a family business might enjoy a compliant entourage and/or a group of executives who pursue the prestige of the family. The rationale for the value of relationships to mature only for senior executives is that relationships take time to be developed.<sup>8</sup>

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<sup>8</sup>Of course, personal relationships might develop before the match forms, for example if an owner hires friends or relatives. In this case, exactly the same logic would apply to "connected" vs. "unconnected" executives. Unfortunately, in the data we have no way of detecting this type of relationships, while we observe seniority. The two channels of personal ties are not mutually exclusive and might both be at play.

We assume that upon hiring a (junior) executive the principal observes neither  $x$  nor  $r$ , but only knows their distribution. At the end of the first period the principal learns the value of the executive's productivity, a realization of  $x$ , and the value of his relationship, the realization of  $r$  (either 0 or  $R > 0$ ). It is assumed that both the executive relationship value and productivity are specific to an executive-firm match, so that if an executive moves to a new firm both his  $x$  and  $r$  are unknown to the new principal. After learning the realizations of  $x$  and  $r$  the principal decides whether to keep the executive in office (i.e., give him tenure) or to replace him with a junior one (i.e., fire the incumbent executive). It is convenient to define a new random variable  $s \equiv x + r$ . Using that  $x$  and  $r$  are independent, the CDF is

$$F(s) = q G(\max(s - R, 0)) + (1 - q) G(s) \quad \forall s > 0 \quad (1)$$

so that the probability that a senior executive with  $r + x \geq s$  is observed is  $1 - F(s)$ .

If appointed, the (senior) executive stays one period with the firm and then dies with an exogenous constant hazard  $\rho$ , so that the expected office tenure of a senior executive is  $1/\rho$ . When a senior executive dies the principal replaces her with a junior one.

The per-period return for the risk-neutral principal is given by the realizations of  $s_t = x_t + r_t$ , his utility is given by the expected present value of the sum of these realizations:  $v \equiv \sum_{t=0}^{\infty} \beta^t s_t$ , where  $\beta$  is a time discount. The principal cares about the executive productivity and his/her relationship value, and decides whether or not to fire an executive after observing the realization of both variables at the end of the first period. When a junior executive is in office at the beginning of period  $t$  there is no further decision to be taken for the principal, and the expected value for the principal is

$$v_y = \mu + \beta \mathbb{E}_{\tilde{s}} \max\{ v_y, v_o(\tilde{s}) \} \quad (2)$$

where expectations are taken with respect to the next-period realization of the executive

value  $\tilde{s}$  and  $v_o(\tilde{s})$  denotes the value of a senior executive with *known* value  $\tilde{s}$ . This value is

$$v_o(\tilde{s}) = \max\{ v_y , \tilde{s} + \beta [ \rho v_y + (1 - \rho) v_o(\tilde{s}) ] \}. \quad (3)$$

where the value function  $v_o(\tilde{s})$  is continuous and non-decreasing in  $\tilde{s}$ .

The optimal policy follows a threshold rule: the principal fires the senior executive if  $s < s^*$ , i.e., if the value of  $s = r + x$ , learned when the executive becomes senior, is below the threshold  $s^*$ . Hence  $s^*$  is the smallest value of  $s = x + r$  that leaves the firm indifferent between keeping the senior executive or appointing a junior one, which solves

$$v_o(s^*) = v_y . \quad (4)$$

Using [equation \(3\)](#) it is straightforward to compute the *expected* value of a senior executive *conditional on being in office* as

$$\begin{aligned} v_o \equiv \mathbb{E}_s(v_o(s)|s > s^*) &= \int_{s^*}^{\infty} s \frac{dF(s)}{1 - F(s^*)} + \beta [\rho v_y + (1 - \rho)v_o] \\ &= \frac{1}{1 - \beta(1 - \rho)} \left[ \int_{s^*}^{\infty} s \frac{dF(s)}{1 - F(s^*)} + \beta \rho v_y \right] . \end{aligned} \quad (5)$$

Given  $s^*$ , the expected value of a junior executive can be rewritten as

$$v_y = \mu + \beta [F(s^*) v_y + (1 - F(s^*)) v_o] \quad (6)$$

Using [equation \(6\)](#) and the expression for  $v_o$  in [equation \(5\)](#) gives a closed form equation for  $v_y$  as a function of  $s^*$ :

$$v_y = \frac{\mu(1 - \beta(1 - \rho)) + \beta \int_{s^*}^{\infty} s dF(s)}{(1 - \beta)[1 + \beta(\rho - F(s^*))]} . \quad (7)$$

Using [equation \(3\)](#) to write the value of a senior executive of type  $s^*$  as

$$v_o(s^*) = \frac{1}{1 - \beta(1 - \rho)} (s^* + \beta\rho v_y)$$

and replacing this expression into [equation \(4\)](#) gives the optimality condition

$$s^* = (1 - \beta)v_y . \tag{8}$$

Using [equation \(8\)](#) and the expression for  $v_y$  in [\(7\)](#) gives one equation in one unknown for  $s^*$ :

$$H(s^*, R) \equiv s^* [1 + \beta(\rho - F(s^*))] - \mu(1 - \beta(1 - \rho)) - \beta \int_{s^*}^{\infty} s dF(s) = 0 . \tag{9}$$

This leads us to:

**PROPOSITION 1.** Given the primitives  $\beta, \rho, G(\cdot), q$ , there exists a unique optimal threshold  $s^*(R)$ . Moreover:

- (i)  $s^*(0) > \mu$
- (ii)  $s^*(R)$  satisfies:

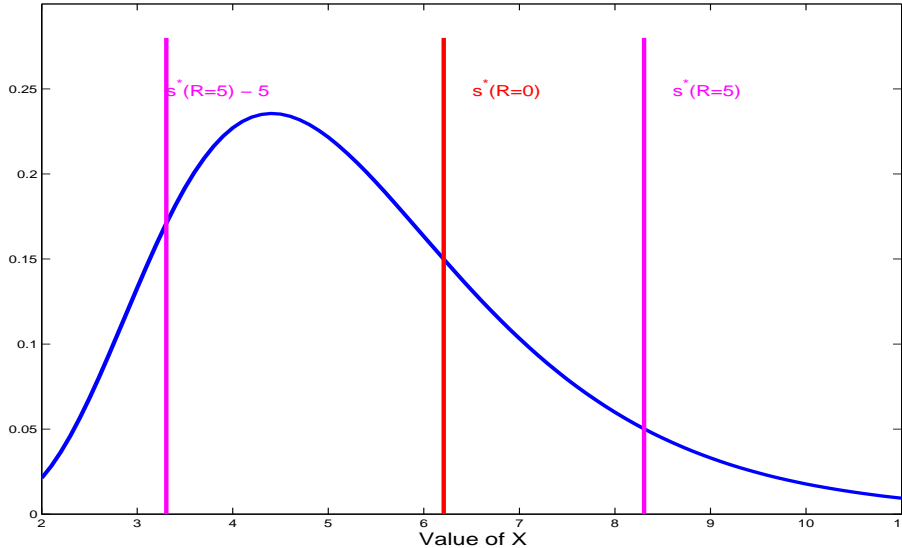
$$0 < \frac{\partial s^*(R)}{\partial R} = q\beta \frac{1 - G(s^* - R)}{1 + \beta(1 - F(s^*))} < q\beta < 1 \tag{10}$$

**Proof.** See [Appendix A](#).

The proposition states that even when  $R \equiv 0$  (relationships bring no value to the principal) the optimal threshold  $s^*(0) > \mu$ . Hence the senior executive retains office only if he is sufficiently above the expected value of a junior  $\mu$ . That is because the appointment of a junior, and the possibility of future replacement, gives the policy of appointing a junior a positive option value. The fact that productivity  $x$  is learned after one period induces a selection whereby senior executives who retain office are more productive than the average junior executive. This is shown in [Figure 1](#) where the optimal threshold for the  $R = 0$  case

lies to the right of  $\mu$  (the mean of the ability distribution).

Figure 1: Example of selection thresholds for  $R = 0$  and  $R = 5$



Note: The figure uses the following parameters:  $\beta = 0.98$  (per year),  $\rho = 0.11$  (per year),  $q = 0.75$ .  $G(\cdot)$  is lognormal with log mean  $\lambda_m = 1.6$ , and log std  $\lambda_\sigma = 0.36$  which imply  $\mu = 5.3$ .

The second part of the proposition characterizes how the optimal threshold  $s^*$  varies with  $R$ . The larger the importance of the non-monetary returns to the principal (as measured by a higher value of  $R$ ), the greater is the value of the threshold  $s^*$ . This has two contrasting effects on the productivity of the executives who get tenure: the fact that  $\partial s^*/\partial R < 1$  implies that, as  $R$  increases, the productivity threshold for the executives who develop a relationship (i.e., those with  $r = R$ ) falls, since  $s^* - R$  is decreasing in  $R$ . On the other hand, the threshold for the executives who do not develop a valuable relationship (i.e., those with  $r = 0$ ) increases: these executives must compensate for their lack of “relationship” value with a higher productivity, such that  $x \geq s^*$ . As shown in [Figure 1](#), the ability thresholds for executives with and without relationship value move apart as  $R$  increases.

We now turn to the model prediction concerning the seniority composition of the firm’s executives in a steady state. The fraction of senior executives in office,  $\phi$ , follows the law of

motion

$$\phi_t = \phi_{t-1}(1 - \rho) + (1 - \phi_{t-1})(1 - F(s^*)) \quad ,$$

so that the steady state fraction of senior executives is

$$\phi(s^*) = \frac{1}{1 + \frac{\rho}{1-F(s^*)}} \in (0, 1) \quad (11)$$

which is decreasing in  $\rho$ . Mechanically, a lower hazard rate increases the fraction of senior executives. It is also immediate that  $\phi$  is decreasing in  $F(s^*)$ . To study how  $\phi$  depends on  $R$  we need to compute the total derivative of  $F(s^*)$ , since changes in  $R$  affect the CDF directly and also affect the threshold  $s^*$ . Note that

$$\frac{dF(s^*)}{dR} = [q g(s^* - R) + (1 - q) g(s^*)] \frac{\partial s^*}{\partial R} - q g(s^* - R). \quad (12)$$

Recalling that  $\frac{\partial s^*}{\partial R} < q$  (see [Proposition 1](#)) shows that the derivative is negative at  $R = 0$ , which means that at  $R = 0$  the share of senior executives is increasing in  $R$ . Intuitively, when  $R > 0$  the appeal of senior executives increases because, all other things equal, their expected return is increased by the expected value of relationships,  $qR$ . However, the effect of  $R$  on  $\phi$  cannot be signed in general when  $R > 0$ . The reason is that an increase in  $R$  has two opposing effects. On the one hand it *lowers* the threshold  $s^* - R$  for that fraction ( $q$ ) of senior executives who display valuable relationships ( $r = R$ ). This increases  $\phi$ . On the other hand a higher  $s^*$  raises the acceptance threshold for the senior executive with no relationship capital ( $r = 0$ ). This reduces  $\phi$ . The final effect thus depends on the features of the distribution of  $x$  and  $r$ . For instance, for sufficiently small values of  $q$  the relationship between  $\phi$  and  $R$  is non-monotone. An example is displayed in the left panel of [Figure 2](#).

We now analyze how changes in  $R$  affect the firm's average productivity in the steady state. Let  $X$  denote the mean productivity of the firm, given by the weighted average of the

expected productivity of the junior and senior executives:

$$X(s^*) = \mathbb{E}_{r,x}(x) = \mu + \phi(s^*) [X_o(s^*) - \mu] \quad (13)$$

where some algebra shows that the senior executives' average productivity is

$$X_o(s^*) = \frac{q \int_{s^*-R}^{\infty} x dG(x) + (1-q) \int_{s^*}^{\infty} x dG(x)}{1 - F(s^*)} . \quad (14)$$

This leads us to:

**PROPOSITION 2.** Let  $\beta \rightarrow 1$ , then the steady state firm productivity  $X(s^*)$  is:

- (i) maximal under the policy  $s^*(R = 0)$ , with  $\frac{\partial X}{\partial R} \Big|_{R=0} = 0$
- (ii) decreasing in  $R$ :  $\frac{\partial X}{\partial R} \Big|_{R>0} < 0$

**Proof.** See [Appendix A](#).

The proposition shows that the mean productivity of a firm is maximized when the firm only cares about ability, i.e., under the policy  $s^*(R = 0)$ . Any policy  $s^*(R)$  with  $R > 0$  induces on average a lower firm productivity. Moreover, the proposition shows that  $X$  is monotone decreasing in  $R$ . This will be useful in the discussion of the parameters identification below. The assumption that  $\beta \rightarrow 1$  simplifies the derivation and is useful to interpret the mean  $X$  as a cross section average.<sup>9</sup> The proposition also establishes that the derivative of  $X$  with respect to  $R$  is zero at  $R = 0$ . This result, and the fact that the share of senior executives is increasing at  $R = 0$  (discussed above) implies that the productivity differential between senior and junior executives,  $X_o - \mu$ , is decreasing in  $R$  at  $R = 0$ , as can be seen from [equation \(14\)](#)

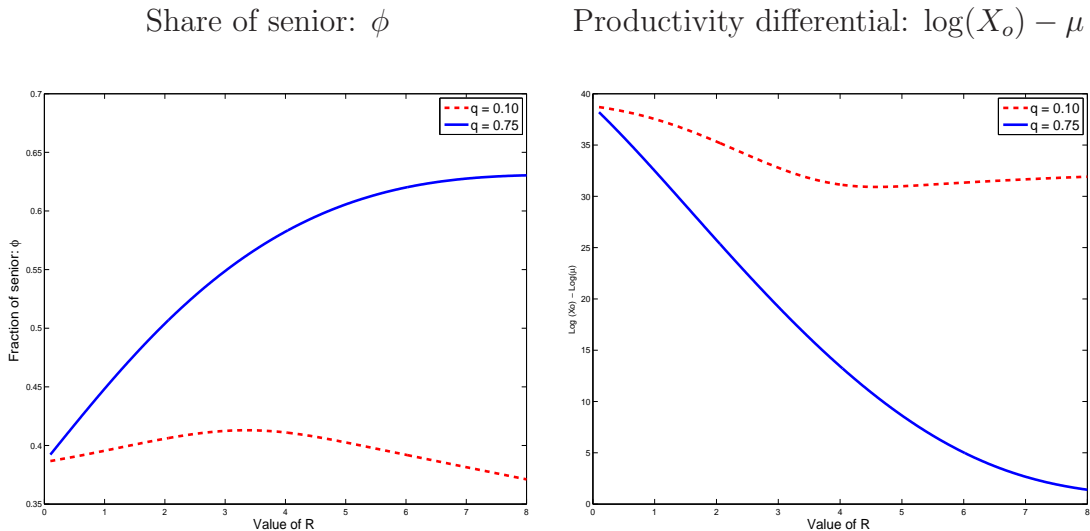
$$\frac{\partial \{X_o(s^*) - \mu\}}{\partial R} \Big|_{R=0} = \frac{g(s^*) \left( q - \frac{\partial s^*}{\partial R} \right) \left[ s^* - \frac{\int_{s^*}^{\infty} x dG(x)}{1 - G(s^*)} \right]}{1 - G(s^*)} < 0 \quad (15)$$

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<sup>9</sup>The numerical analysis of the model for  $\beta \in (0.85, 1)$  (per year) gives very similar results. The relationship between  $X$  and  $R$  is always decreasing.

The intuition behind this pattern is simple: as  $R$  increases the owner selects less on ability, so that the senior executives become more similar to the unselected pool of junior executives. The right panel of [Figure 2](#) shows that this pattern holds for a wide set of parameter values and, in particular, for the parameters that are in a (broad) neighborhood of our structural estimates (thick line). The figure also shows that in the parametrization with the high value of  $q = 0.75$ , both  $\phi$  and  $X_o - \mu$  become flat functions of  $R$  for  $R \cong 8$ : at this point owners are basically already firing all executives with  $r = 0$  and keeping all those with  $r = 8$ , so that further increases in  $R$  do not influence the selection process anymore. In other words,  $X$  and  $\phi$  asymptote to constant values as  $R$  grows large. We will come back to this observation when we comment on the results of our estimates.

Figure 2: Share of Senior executives and Senior-Junior differential as  $R$  varies



Note: The figure uses the following parameters:  $\beta = 0.98$  (per year),  $\rho = 0.11$  (per year),  $q = 0.75$ .  $G(\cdot)$  is lognormal with log mean  $\lambda_m = 1.6$ , and log std  $\lambda_\sigma = 0.36$  which imply  $\mu = 5.3$ .

## 4 Identification

We now discuss the mapping between model and data and, in particular, the data variability that identifies the model's parameters. We first show how the structural parameters can be



identified by the average  $X, \phi$  that characterize an owner with a given  $R, q$ . Then we show that the model also delivers a restriction that has a very natural interpretation in terms of an OLS regression. This regression identifies a subset of the model's parameters that can be compared with those of the structural parameters' estimates. These two exercises exploit two completely different dimensions of data variability. The results therefore can be compared to gain some insights on the robustness of our findings.

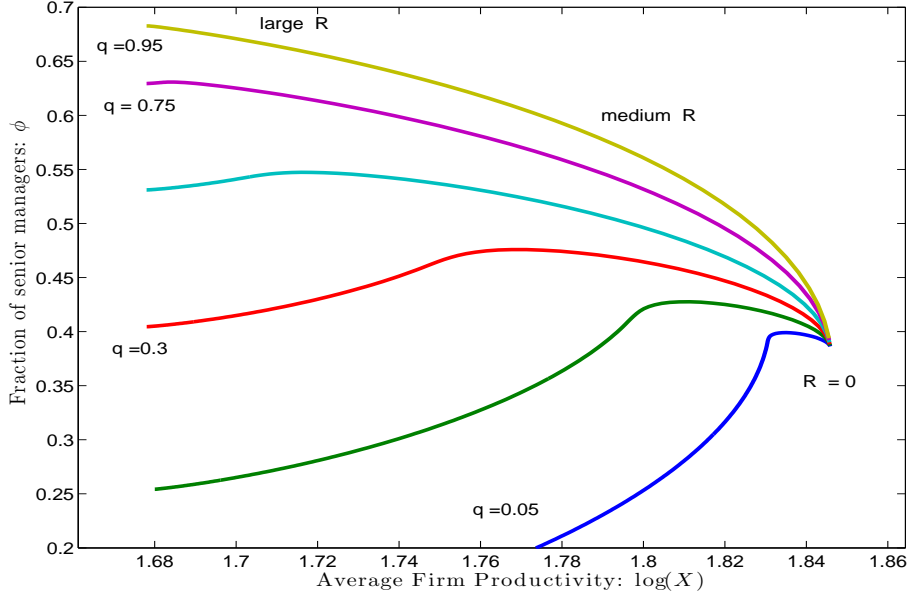
The model yields predictions concerning two observable variables: the firm productivity  $X$  and the fraction of senior executives  $\phi$ . For a given vector of model primitives  $\beta, \rho, G(\cdot)$ ,<sup>10</sup> **Figure 3** shows that for each admissible (i.e., model generated) observable pair  $X, \phi$  there is at most one pair of parameter values  $R, q$  that can produce it. Each line in the figure is indexed by one value of the parameter  $q$ . Increasing  $q$  shifts the locus upward: a higher probability of maturing a valuable relationship increases the likelihood of being tenured and hence  $\phi$ . Notice that all lines depart from the same point in the  $X, \phi$  plane, which corresponds to  $R = 0$ . This point corresponds to the productivity maximizing situation, obtained when relationships have no value. Starting from this point, an increase in  $R$  moves the model outcomes along one line (indexed by  $q$ ) from right to left. We know this because, as shown in **Proposition 2**,  $X$  is decreasing in  $R$ . Moreover, as discussed above, the effect of an increase in  $R$  on  $\phi$  is, in general, not monotone. This explains why the lines that correspond to low values of  $q$  are hump-shaped. The important point of this figure is that those lines never cross, so that given any point in the space spanned by the model, one can invert it and retrieve the values of  $q$  and  $R$  that produced it.

We now discuss another implication of the theory that can be exploited to estimate the productivity differential between the senior and the junior executives. Consider a set of firms drawn from a given model parametrization. Firm  $i$  employs  $n_i$  executives. Firms differ with respect to the quality of executives, which depends on the realizations of  $x + r$  for each executive. The econometrician observes the firm's productivity  $X_i$  and the fraction of senior

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<sup>10</sup>We discuss in the next section how we pin down these objects.

Figure 3: Productivity and Seniority: space spanned by the model



Note: The figure uses the following parameters:  $\beta = 0.98$  (per year),  $\rho = 0.11$  (per year).  $G(\cdot)$  is lognormal with log mean  $\lambda_m = 1.6$ , and log std  $\lambda_\sigma = 0.36$  which imply that  $\log(\mu) = 1.67$ .

executives  $\phi_i$ . Let  $X_o$  and  $X_y = \mu$  be the mean productivity of the incumbent senior and junior executives, respectively. Actual productivities differ from the mean ones because of sampling variability. Using equation (13) we establish the following proposition:

**PROPOSITION 3.** The productivity of firm  $i$  can be written as:

$$X_i = \mu + (X_o - \mu) \phi_i + \varepsilon_i \quad (16)$$

where  $\mathbb{E}\{\varepsilon_i\} = 0$  and  $\mathbb{E}\{\phi_i \varepsilon_i\} = 0$ .

**Proof.** See Appendix A.

A key result from this proposition is that deviations  $\varepsilon_i$  about the (large sample or unconditional) mean values are uncorrelated with the share of senior executives  $\phi_i$ . Intuitively, this property holds since an increase (or a decrease) in the quota of senior executives  $\phi_i$  about its unconditional mean  $\phi$  does not contain any information on the innovation  $\varepsilon_i$ , i.e., the amount by which the productivity of the senior (junior) executive exceeds the selection threshold  $s^*$

in firm  $i$ . This result implies that the productivity differential  $X_o - \mu$  can be estimated with an OLS regression of  $X_i$  on  $\phi_i$ . Compared to the structural estimates this regression only identifies the productivity differential and not directly the values of  $R, q$ . These estimates however are useful for at least two reasons. First, they supply an independent measure of one key prediction of the model. In fact, this estimate is based on a totally different dimension of data variability. As discussed above, the structural estimates use the average values of  $X, \phi$  of a given control-type to back out the  $R, q$  pair consistent with them. Instead, the OLS estimates exploit the partial correlation coefficient between  $X$  and  $\phi$  across firms for a given  $R, q$ . Therefore, they do not depend on the average  $X, \phi$  for a given control type, but on how  $X$  and  $\phi$  co-vary across firms of a given type. The intuition is the following. When selection is weak (i.e.,  $R$  is large), two firms with different shares of senior executives differ little in productivity, since on average senior executives are not much more productive than junior executives. This implies that the correlation between  $X$  and  $\phi$  is low. If  $R$  is low, differences in  $\phi$  will go together with substantial differences in  $X$ , yielding a high correlation between  $X$  and  $\phi$ . Furthermore the difference will be larger the stronger the selection. A second important feature of the OLS estimates is that they do not require one to pin down all the structural parameters. In particular, they are independent of (and, of course, do not supply any information on)  $\beta, q, \rho, G(\cdot)$ . They therefore offer a test of robustness of the structural estimates with respect to the values of the auxiliary parameters they hinge upon.

## 5 Data description

In this section we describe the main features of our data, referring to [Appendix B](#) for more details. The data match a large sample of executives with a sample of Italian firms. The firm data are drawn from the Bank of Italy’s annual survey of manufacturing firms (INVIND), an open panel of around 1,200 firms per year representative of manufacturing firms with at least 50 employees. It contains detailed information on firms’ characteristics, including industrial

sector, year of creation, number of employees, value of shipments, value of exports and investment. It also reports sampling weights to replicate the universe of firms with at least 50 employees. We completed the dataset with balance-sheet data collected by the Company Accounts Data Service (CADS) since 1982, from which it was possible to reconstruct the capital series, using the perpetual inventory method.

Our measure of productivity is TFP. We assume that production takes place with a Cobb-Douglas production function of the form:

$$Y_{it} = \text{TFP}_{it} K_{it}^{\beta} L_{it}^{\alpha} \quad (17)$$

where  $Y$  is value added,  $K$  is capital and  $L$  labor. TFP depends on average managerial ability  $X$  and, possibly, on other additional observable and unobservable characteristics  $W_{it}$ , such as time, sector and firm size:

$$\text{TFP}_{it} = \left( \frac{1}{n_{it}} \sum_{j=1}^{n_{it}} X_{j \in i,t} \right) * e^{W_{it} + \epsilon_{it}} \quad (18)$$

where  $n_{it}$  is the number of executives in firm  $i$  at  $t$ ,  $X_{j \in i,t}$  is the ability of executives in firm  $i$  at  $t$  and  $\epsilon_{it}$  is an iid shock unobserved to the firm or, more simply, measurement error in TFP. We estimate TFP using the [Olley and Pakes \(1996\)](#) approach. The procedure is briefly described in [Appendix B](#); full details are in [Cingano and Schivardi \(2004\)](#).

The survey contains several questions regarding the controlling shareholder. The most relevant for our purpose is “What is the nature of the controlling shareholder?”, from which we construct an indicator that groups firms into one of four control categories (see [Appendix B](#) for the details): 1) individual or family; 2) government (local or central or other government controlled entities); 3) conglomerate, that is, firms belonging to an industrial conglomerate; 4) institution, such as banks and insurance companies, and foreign owners. We expect these different types of ownership to be characterized by different degrees of relevance of personal relationships. Owners of family business are likely to derive utility from controlling the firm

above and beyond the pure monetary returns.<sup>11</sup> Part of these returns might come from a compliant entourage and/or a group of executives who pursue the prestige of the family. A politician (the “owner” of a government controlled firm) might want executives who serve his political interests, such as hiring workers in his constituency. We therefore expect these type of firms to be characterized by positive values of  $R$ . Firms controlled by other entities, such as a foreign institution or a conglomerate, are instead likely to put more weight on pure monetary returns.<sup>12</sup> Independently from these presumptions, in the estimation exercise we will not put any restriction on the values of  $R$  and will let the data speak. In fact, the literature shows that personal ties are pervasive and their effects important even in large, listed firms (Landier, Sraer, and Thesmar, 2006; Kramarz and Thesmar, 2006), so that they might also be relevant for conglomerate or foreign firms. Moreover, some firms that declare to be controlled by an institutional owner might in reality be family firms, with the family exercising control through an intermediate body. Notice that, in this case, according to our presumptions we would be over-estimating the relevance of personal ties in institution controlled firms so that the estimated difference between this category and family firms would under-estimate the actual difference.

Table 1 reports summary statistics for the firm data used in the regression analysis both for the total sample and by control type. For the total sample, on average, firms have value added of 30 million euros (at 1995 prices) and employ 691 workers of which 13 executives. The average ratio of executives to total workforce is 2.6%. Around 41% of firms are classified as medium-high and high-tech according to the OECD 2003 system and 3/4 are located in the north. Clear differences emerge according to the control type. Family firms are substantially smaller than the average (11 million euros and less than 300 employees) and specialize in more traditional activities. Importantly, they have a lower TFP level, followed by government

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<sup>11</sup>See the literature cited in Section 2.

<sup>12</sup>We lump institutional and foreign owners together because both ownership types are not likely to be identifiable with a single individual, so that from our perspective it makes sense to assume a common  $R$ . Moreover, these two types by themselves have substantially fewer observations than family or conglomerate firms (see Table 1), making inference less reliable. We have experimented with five categories, distinguishing between foreign and institutions, finding similar (although less precise) results.

Table 1: Descriptive statistics: firms' characteristics, by Control type

	V.A.	Empl.	# Exec.	% Exec.	TFP	% High Tech	% North	N. obs.
<b>All firms</b>								
Mean	30.0	692	13.3	0.026	2.41	0.41	0.74	7,773
S.D.	127.3	3,299	29.1	0.021	0.51	0.49	0.44	
<b>Family</b>								
Mean	11.2	281	5.7	0.024	2.33	0.33	0.73	2,906
S.D.	18.0	420	9.8	0.016	0.46	0.47	0.44	
<b>Conglomerate</b>								
Mean	44.5	1024	15.0	0.026	2.44	0.40	0.82	2,390
S.D.	214.2	5,637	27.5	0.025	0.54	0.49	0.39	
<b>Government</b>								
Mean	40.6	1,013	21.8	0.022	2.38	0.47	0.51	687
S.D.	87.4	2,076	57.5	0.021	0.61	0.50	0.50	
<b>Foreign</b>								
Mean	37.0	791	19.8	0.030	2.53	0.52	0.75	1,790
S.D.	69.1	1,563	32.9	0.022	0.48	0.50	0.44	

NOTE: V.A. is value added (in millions of 1995 euros), # Exec. is the number of executives, % Exec. is the share of executives over the total number of employees, TFP is total factor productivity, High Tech is the share of firms classified as medium-high and high tech according to the OECD classification system (OECD, 2003), North is the share of firms located in the North, N. obs. is the number of firm-year observations.

controlled firms, while foreign firms have the highest TFP.

Table 2: Descriptive statistics: executives' characteristics, by Control type

	Wage	$\phi_5$	$\phi_7$	Age	Male
<b>All firms</b>					
Mean	1,236	0.57	0.45	46.5	0.96
S.D.	330	0.30	0.31	4.6	0.13
<b>Family</b>					
Mean	1,130	0.62	0.51	46.0	0.94
S.D.	288	0.33	0.33	5.3	0.17
<b>Conglomerate</b>					
Mean	1,294	0.53	0.41	46.6	0.97
S.D.	321	0.28	0.28	4.0	0.09
<b>Government</b>					
Mean	1,298	0.54	0.42	47.7	0.99
S.D.	349	0.27	0.27	4.6	0.03
<b>Foreign</b>					
Mean	1,309	0.54	0.43	46.9	0.96
S.D.	361	0.29	0.29	4.1	0.12

NOTE: Wage (gross, per week) is in 1995 euros.  $\phi_5$  the share of executives with at least 5 years of seniority,  $\phi_7$  with at least 7 years. Age is the average executives' age and Male is the share of male executives.

The executives' data are taken from the Social Security Institute (Inps), which was asked to provide the complete work histories of *all* workers who were ever employed in an INVIND firm over the period 1981-1997. Workers are classified as blue collar (*operai*), white collar (*impiegati*) and executives (*dirigenti*). The data on workers include age, gender, area where the employee works, occupational status, annual gross earnings, number of weeks worked and the firm identifier. We only use workers classified as executives. In our preferred specification an executive turns senior after five years of tenure. [Table 2](#) reports the statistics on executives'

characteristics for the total sample and by control type. For the total sample, average gross weekly earnings at 1995 constant prices are 1,236 euros, the share of executives that have been with the firm at least five years is .57 and at least seven years .45. Executives are on average 46.5 years old and 96% are male. Family controlled firms pay lower wages to their executives and have a higher share of senior executives (62%). Executives' characteristics at conglomerate controlled firms are fairly similar to the overall ones. Government controlled firms employ older and almost exclusively male executives. Finally, foreign control firms pay their executives more, while their executive's resemble the average in terms of the tenure, age and gender composition.

## 6 Structural estimates

This section describes the structural estimation of the model. We begin by discussing the assumptions needed for the estimation of the model parameters using firm level observations on TFP and seniority of the executives, i.e., what we see as empirical measures of  $X$  and  $\phi$ . The exercise assumes the data are generated by the model and observed with classical measurement error. The estimation is developed under the assumption of observed heterogeneity, as some structural parameters are linked to observable characteristics of the firm, along the lines of [Alvarez and Lippi \(2009\)](#). Assuming that the distribution of productivity  $G(x)$  is lognormal, the model is characterized by six fundamental parameters: the discount factor  $\beta$ , the hazard rate  $\rho$ , the lognormal parameters  $\lambda_m, \lambda_\sigma$  (log mean and log standard deviation, respectively), the probability developing a relationship,  $q$ , and the value of the relationship  $R$ . Five parameters, namely  $\beta, \rho, q, \lambda_\sigma$  and  $\lambda_m$ , are assumed common to all firms. The parameter  $R$  is assumed to vary with one observable characteristic of the firm: the control type. Given a parametrization, the model uniquely determines the values of  $X, \phi$  to be observed in the data. Differences between datapoints with identical observables (e.g. two firms with the same control type) are accounted for by classical measurement error. Next,



we fill in the details that relate to the data used in estimation, and describe the estimation algorithm and results.

Our parsimonious structural estimate concerns six parameters:  $\theta_p \in \Theta_{6,1}$ ,  $p = 1, 2, \dots, 6$ , where  $\theta_6$  gives the probability of developing a relationship  $q = \frac{\theta_6}{1+\theta_6}$ . The firm-level observations vary across 14 years (index  $t$ ), 13 two-digit sectors (index  $\tau$ ), and four control types described in the previous section (index  $\kappa$ ). We assume that  $R$  varies across firms according to the nature of the controlling shareholder, with  $R_\kappa = \theta_\kappa$ ,  $\kappa = 1, 2, 3, 4$  for the firms under, respectively, family ( $\theta_1$ ), conglomerate ( $\theta_2$ ), government ( $\theta_3$ ), and foreign ( $\theta_4$ ) control. The technological parameter  $\lambda_m = \theta_5$  is related to the mean TFP of the firm. In the data TFP has a clear time component, as well as a sectoral one, that are ignored by the simple structure of our model. Thus, before turning to the structural estimation, we normalize the TFP data by removing common time and sector effects. Our measure of  $X$  for firm  $i$  in year  $t$  and sector  $\tau$  is thus given by

$$\log X_{i,t,\tau} \equiv \log TFP_{i,t,\tau} - \mathbf{a}_1 \cdot \mathbf{year}_{i,t} - \mathbf{a}_2 \cdot \mathbf{sect}_{i,\tau} - a_3 \cdot Z_{i,t,\tau} \quad (19)$$

where  $\mathbf{a}_1$  and  $\mathbf{a}_2$  are the vector of coefficients from an OLS regression of TFP on 13 year and 12 two digit sector dummies. Below we also consider a specification that controls for the effect of firm size  $Z_{i,t,\tau}$  (log employment) on TFP.

To reduce the computational burden some parameters are pinned down outside the estimation routine. We calibrate the time discount  $\beta$  to an annual value of 0.98, as standard in the literature. The hazard rate of senior executives,  $\rho$ , is computed from the survival function of senior executives, that is with at least five years of seniority, using the [Kaplan and Meier \(1958\)](#) estimator on the individual data. We estimate  $\rho = 0.11$  per year, which implies that the expected tenure of senior executives is approximately 10 years. The variance of the talent distribution is computed using the junior executives' compensation data. The idea is that junior executives' compensation, being independent from the private benefits  $R$ , reflects on

average the executive’s individual ability.<sup>13</sup> We therefore regress the log wage on age, age squared, firm size (log of the employees) and dummies for years, sectors, control type and seniority (from 1 to 5) and take the standard deviation of the residuals as our measure of the standard deviation of the ability distribution. We find a value  $\lambda_\sigma = 0.36$ , which varies very little with respect to changes in the set of controls. The sensitivity of the structural estimates to the values of  $\lambda_\sigma$  is discussed at the end of this section.

These assumptions imply that, after removing time and sectoral differences, all firms in a group –indexed by the control type  $\kappa = 1, 2, 3, 4$ – are expected to have the same  $X$  and  $\phi$ . For each firm  $i$  in group  $\kappa$  there are two observables  $y_{i,\kappa}^j$ ,  $j = 1, 2$ . We assume that the variable  $y_{i,\kappa}^j$  is measured with error  $\varepsilon_i^j$  that is normal, with zero mean, independent across variables, groups and observations. Inspection of the raw data suggests that measurement error is multiplicative in levels for TFP,  $X$ , and additive for the share of senior executives,  $\phi$ .<sup>14</sup> Hence the ML estimates use the following observables  $y_{i,\kappa}^1 = \log X_{i,\kappa}$  and  $y_{i,\kappa}^2 = \phi_{i,\kappa}$ . The measurement error variances  $\sigma_j^2$ ,  $j = 1, 2$  is assumed common across groups, and is computed as the variance of the residuals of an OLS regression of  $\log X$  and  $\phi$  on year and sector dummies. This gives  $\sigma_{\log X}^2 = 0.35$  and  $\sigma_\phi^2 = 0.29$ .

Let  $f^j(\Theta, \kappa)$  be the model prediction for the  $j^{\text{th}}$  variable in group  $\kappa$  under the parameter settings  $\Theta$ . The observation for the corresponding variable for firm  $i$  in group  $\kappa$  is

$$y_{i,\kappa}^j = f^j(\Theta, \kappa) + \varepsilon_i^j.$$

Let  $Y$  be the vector of observations and  $n_\kappa$  be the number of firms  $i$  in group  $\kappa$ . Define the

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<sup>13</sup>In the model, ability is revealed abruptly when an executive turns senior. In theory, therefore, all junior executives should be paid the same wage, as the owner has no information on ability. Of course, in reality the process of learning about executives skills is more gradual, so that the wage over the junior period does convey some information about individual ability.

<sup>14</sup>This statement is based on an analysis of the deviations of  $X$  and  $\phi$  (in levels and in logs) from the mean of each groups. Details are available from the authors upon request.

objective function  $F$  as

$$F(\Theta; Y) \equiv \sum_{\kappa=1}^4 \sum_{j=1}^2 \left( \frac{n_{\kappa}}{\sigma_j^2} \right) \left( \frac{\sum_{i=1}^{n_{\kappa}} y_{i,\kappa}^j}{n_{\kappa}} - f^j(\Theta, \kappa) \right)^2 \quad (20)$$

Appendix C shows that the likelihood function is related to the objective function by:

$$\log L(\Theta; Y) = -\frac{1}{2} \sum_{\kappa=1}^4 \sum_{j=1}^2 n_{\kappa} (1 + \log(2\pi\sigma_j^2)) - \frac{1}{2} F(\Theta; Y)$$

We estimate the six structural parameters in  $\Theta$  by minimizing (20). At each iteration, the algorithm solves the model for each of the four groups and computes the objective function under a given parametrization. Since each group has two observables there is a total of eight moments to be fitted using 6 parameters, hence the model is over-identified with two degrees of freedom. The formulas for the score and the information matrix used for the inference are derived in Appendix C.

Table 3: Structural estimates of model parameters

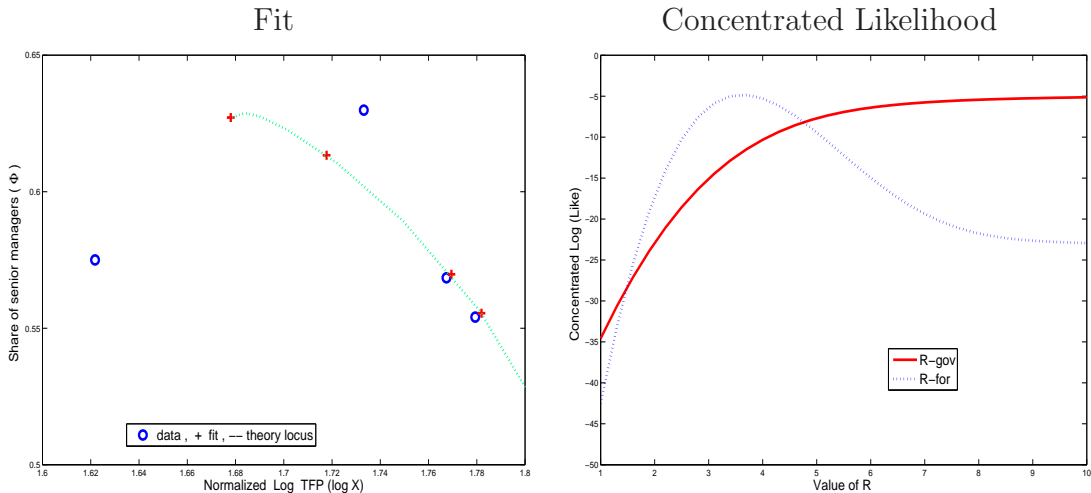
	$q(\theta_6)$	$R_{fam}(\theta_1)$	$R_{cong}(\theta_2)$	$R_{gouv}(\theta_3)$	$R_{forgn}(\theta_4)$	$\lambda_m(\theta_5)$
Baseline	0.74 (5.6)	5.6 (10.1)	3.2 (12.5)	41.4 (0.0)	3.6 (10.0)	1.61 (173)
B1: w. firm-size	0.72 (5.7)	5.1 (10.3)	3.4 (11.8)	86.9 (0.0)	3.7 (8.9)	1.61 (166)

Note: t-statistics in parenthesis. The estimation assumes  $\sigma_{\log X}^2 = 0.35$  and  $\sigma_{\phi}^2 = 0.29$ . The parameters  $\beta, \rho, \lambda_{\sigma}$  are fixed from an auxiliary estimation (see the main text). The measure for productivity  $\log X$  is the firm level (log) TFP net of the common components due to year effects and sector effects, as from equation (19) (see the main text); in the specification B1 firm-size effects (as measured by the the (log) number of employees) are also controlled for.

The structural estimates of the model parameters are reported in Table 3. The estimated value of  $q = 0.74$  indicates that approximately 3/4 of the executives develop a relationship. The estimates show that the values for  $R$  vary substantially across control control-types, but that all types enjoy some degree of private benefits. This is consistent with the findings of Taylor (2010), according to which CEO entrenchment is substantial even in US listed firms,

where family and government firms play a minor role.<sup>15</sup> The importance of relationship is lowest for firms belonging to a conglomerate (3.2), followed by foreign controlled firms (3.6), family (5.6), and government (41.4). Given that the estimated unconditional mean level of TFP is around 7, the estimated values for  $R$  show that the non-monetary characteristics of the executives (i.e., their relationship value) are quantitatively important in the selection process. The values are all strongly statistically significant but for the government case. The reason for the lack of significance for the coefficient of government-controlled firms is revealed by the analysis of the likelihood function in Figure 4.

Figure 4: Fit of estimates



Note: This figure uses the parameters reported in the baseline estimate of Table 3.

The left panel of the picture shows that the observations to be fitted for the government control group are outside the space that can be spanned by the model in the  $X$  dimension. The TFP level for government-controlled firms is below  $\mu$ , the unconditional mean value of  $X$ . In attempting to fit such a low value the model uses a high value of  $R$  (which reduces  $X$  by Proposition 2). Notice that the model has an asymptote: once  $R$  is so large that the selection threshold  $s^* - R$  hits zero, then we have that  $X \cong \mu$ . There is no selection because senior executives are retained in office if and only if  $r = R$ , independently from their ability,

<sup>15</sup>Alternatively, as discussed above, some of the firms that we classify as conglomerate or foreign might in reality be indirectly controlled by a family or an individual.

and their average ability equals that of an unselected pool of junior executives. This is the lowest possible value achievable by the model. This effect is clearly seen from the analysis of the concentrated likelihood function reported in the right panel of the figure.<sup>16</sup> The picture clearly shows that in the estimation of the government  $R = \theta_3$  the concentrated likelihood function (the thick red line) becomes virtually flat for values of  $\theta_3 \geq 8$ . The null hypothesis that  $\theta_3 = 0$  is strongly rejected at the 1 per cent confidence level (as the large differences in the value of the log likelihood on the vertical axis indicate). This evidence shows that the value of  $R$  for the government controlled firms is the largest possible, although the ML estimate is not able to identify a single value of  $R$  in the range  $(8, +\infty)$ . This is in stark contrast with, e.g., the estimate for  $R$  in foreign controlled firms, where the concentrated likelihood is single peaked (dashed blue line) around 3.6 (the ML estimate of [Table 3](#)).

Table 4: Model predictions under the benchmark estimates

	Control type				
	$R = 0$	Family	Conglomerate	Government	Foreign
R	0	5.58	3.21	41.4	3.62
$s^*$	6.21	8.61	7.39	29.2	7.59
$x^*$	6.21	3.03	4.18	0	3.96
$\log \mu$	1.68	1.68	1.68	1.68	1.68
$\log X$	1.85	1.72	1.78	1.68	1.77
$\log X_o$	2.07	1.74	1.86	1.68	1.83
$\log X_o R = 0$	2.07	2.32	2.2	3.55	2.22
$\log X_o R > 0$	2.07	1.72	1.83	1.68	1.81
$\phi$	0.39	0.61	0.56	0.63	0.57
Fired	0.72	0.30	0.45	0.26	0.41
Fired $ R = 0$	0.72	0.93	0.86	1	0.87
Fired $ R > 0$	0.72	0.080	0.31	0	0.26

The table reports key statistics for the steady state of our model solved using the benchmark estimates of [Table 3](#).  $\log X$  is the (log) average managerial ability,  $\log X_o$  is the average managerial ability of the senior executives,  $\log X_o|R = 0$  is the average ability of the senior that did not develop a relationship,  $\log X_o|R > 0$  for those that did develop a relationship. Fired is the probability that a junior executive is replaced when turning senior.

<sup>16</sup>This is computed by evaluating the likelihood function along the parameter of interest ( $\theta_4$  for the government  $R$ ) freezing all other parameter estimates at their optimal values  $\theta^*$  reported in [Table 3](#).

For a better grasp of the implications of the estimates, in [Table 4](#) we report some statistics from the model steady state produced by the baseline estimates of [Table 3](#). The first column uses the model to compute some statistics for the case in which the firm’s principal gives no value to relationships in executive selection ( $R = 0$ ) and hence  $s^* = x^*$  for all senior executives. In this case, senior executives are confirmed if their ability  $x$  is above  $x^* = 6.21$ , which occurs in around 25% of cases. The (log) average ability of senior executives is 2.07, almost 30 log points higher than the unconditional average ability of junior ones. On average, around 40% of executives are senior. Things are different for family firms, for which  $s^* = 8.61$  and  $s^* - R = 3.01$ , where the latter is the cutoff ability of executives that have developed a relationship with the owner. With these values, the selection is much weaker: the senior executives’ average ability is only 6% higher than that of junior executives. An executive who develops a relationship has a 92% chance of being tenured. This probability drops to 7% for a senior executive who does not develop a relationship. For conglomerate and foreign controlled firms, the situation is intermediate between the  $R = 0$  case and the family case. For government firms, instead, the estimates imply that selection occurs exclusively on the basis of developing a relationship: all executives with  $r = R$  are retained, all others are fired. As a consequence, the average ability of senior executives is identical to that of junior executives: the selection effect is completely inhibited.

We have performed some robustness checks. First, we saw in [Table 1](#) that average firm size differs across control types. We therefore re-estimated equation (19) also including firm size among the determinants of TFP. We estimated the model with this measure of ability. The results are reported in the lower panel of [Table 3](#) and are similar to those without firm size. Another important parameter relates to the standard deviation of managerial ability. To check the sensitivity of the results to this parameter, [Table 5](#) reports the results of two alternative estimation exercises that use two different values of  $\lambda_\sigma$ , equal respectively to 0.5 and 1.5 times the value used in the baseline estimates. First, the table shows that the estimated patterns for  $R$  are robust. In all exercises government and family controlled firms

have the largest values of  $R$ , while the enterprises controlled by conglomerates are the most efficient (smallest  $R$ ). Moreover, the effects of selection are more important the higher is the dispersion of ability. The differences in the ability of junior and senior executives increase with  $\lambda_\sigma$  (conditional on performing some selection). Naturally, when ability is very dispersed selection is very effective in increasing productivity. Second, a higher value of  $\lambda_\sigma$  increases the level of the estimated  $R$ .<sup>17</sup> The reason is simple: increasing the dispersion level enhances the effect of selection. Thus an increase in the variance of ability – keeping the mean constant – allows the principal to achieve a much better average productivity. A larger value of  $R$  is thus necessary to offset this force and keep the mean productivity predicted by the model aligned with the data. Third, the share of senior executives changes only marginally for the different values of  $\lambda_\sigma$ . This shows that our results are robust to different parameterizations of  $\lambda_\sigma$ .

## 7 Model-based OLS regressions

In this section we exploit [Proposition 3](#) to construct an OLS based estimate of the productivity differential between senior and junior executives. [Equation \(16\)](#) establishes a relationship between the share of senior executives and firm level TFP. By taking the log of both sides and applying a first order Taylor series approximation around  $\mu$ , we obtain:

$$\log X_i = \gamma_0 + \gamma_1 \phi_i + \eta_i \tag{21}$$

where  $\gamma_0 = (1 + \log \mu)$  and  $\gamma_1 = \frac{X_\sigma - \mu}{\mu}$  and where, as shown in [Proposition 3](#),  $\eta_i$  is uncorrelated with  $\phi_i$ . This equation shows that, in a regression of log TFP on the share of senior executives, the coefficient  $\gamma_1$  measures the percentage difference in average ability between

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<sup>17</sup>The exception is for family firms, where  $R$  is higher in the low  $\lambda_\sigma$  case. This is because the estimate in this case has reached the asymptote, as was the case for the government firms in the baseline estimates. This is apparent from the fact that selection is totally absent for family firms (as well as for government firms) for this parametrization.

Table 5: Estimation results: sensitiveness with respect to  $\lambda_\sigma$

	Control type				
	$R = 0$	Family	Conglomerate	Government	Foreign
	$\lambda_\sigma = .18$				
R	0	16.2	1.59	13.3	1.88
$s^*$	6	14.8	6.59	13.2	6.73
$x^*$	6	0	5.01	0	4.85
$\log \mu$	1.72	1.72	1.72	1.72	1.72
$\log X$	1.8	1.72	1.77	1.72	1.77
$\log X_o$	1.91	1.72	1.82	1.72	1.8
$\log X_o R = 0$	1.91	2.74	1.98	2.63	2
$\log X_o R > 0$	1.91	1.72	1.8	1.72	1.79
$\phi$	0.41	0.62	0.55	0.62	0.57
Fired	0.69	0.28	0.45	0.28	0.42
Fired $ R = 0$	0.69	1	0.85	1	0.87
Fired $ R > 0$	0.69	0	0.31	0	0.25
	$\lambda_\sigma = .54$				
R	0	7.28	5.1	116	5.54
$s^*$	6.51	9.44	8.33	70.9	8.54
$x^*$	6.51	2.16	3.23	0	3
$\log \mu$	1.65	1.65	1.65	1.65	1.65
$\log X$	1.9	1.72	1.79	1.65	1.77
$\log X_o$	2.23	1.76	1.88	1.65	1.85
$\log X_o R = 0$	2.23	2.52	2.42	4.54	2.44
$\log X_o R > 0$	2.23	1.71	1.83	1.65	1.8
$\phi$	0.36	0.61	0.56	0.62	0.57
Fired	0.75	0.32	0.44	0.28	0.41
Fired $ R = 0$	0.75	0.91	0.87	1	0.88
Fired $ R > 0$	0.75	0.09	0.27	0	0.22

The table reports the key statistics from the model estimated under two values of  $\lambda_\sigma$ . The choice of the auxiliary parameters is the same as in [Table 3](#).  $\log X$  is the average managerial ability,  $X_o$  is the average managerial ability of the senior executives,  $\log X_o|R = 0$  is the average ability of the senior that did not develop a relationship,  $\log X_o|R > 0$  for those that did develop a relationship. Fired is the probability that a junior executive is replaced when turning senior.

senior and junior executives. This parameter can be compared with the difference implied by the structural estimates of [Table 4](#). As argued above, the OLS estimates are based on a different dimension of data variability, that is the cross-firm correlation between TFP and



the share of senior executives within control group. This implies that they are robust to potential differences in either average TFP or seniority structure that, on average, equally affects all firms in a control type. For example, if foreign controlled firms tend to attract more junior executives than the other types, this would not affect the OLS estimates while it would clearly affect the structural estimates. Moreover, the OLS estimates do not require any of the auxiliary parameters used in the structural estimation. Therefore, we see these estimates as an important check of the results of the previous section.

We implement [equation \(21\)](#) with the following specification:

$$\log TFP_{it} = \gamma_0 + \gamma_1 \phi_{it} + \gamma_2 D_f \cdot \phi_{it} + \gamma_3 D_g \cdot \phi_{it} + \gamma_4 D_p \cdot \phi_{it} + \gamma_5 \mathbf{W}_{it} + \epsilon_{it} \quad (22)$$

where  $D_i$ ,  $i = f, g, p$  are control status dummies equal to one for family, government and foreign controlled firms respectively, and  $\mathbf{W}_{it}$  is the same vector of controls as in the structural estimates (year dummies, 2-digit sector dummies). We also include the control status dummies, that account for potential unobserved heterogeneity across firms with different control types. We run a unique regression rather than separate ones by control type to maximize comparability with the structural estimates, where the additional controls were introduced in a unique regression as well. The coefficient  $\gamma_1$  measures the percentage difference in the average ability of senior and junior executives in conglomerate controlled firms. Given that the productivity of junior executives is the same across groups,  $\gamma_k$ ,  $k = 2, 3, 4$  represents the difference in the average ability of senior executives for the corresponding control type with respect to conglomerate firms:  $\gamma_k = X_o(R_k) - X_o(R_{\text{conglomerate}})$ .

According to our prior beliefs, as well as to the structural estimation results, we expect that  $\gamma_k < 0$ . This is exactly what we find in [Table 6](#). To obtain population consistent estimates, unless otherwise specified we weight observations with population weights, available from the INVIND survey. In the column labeled ‘‘Structural’’ we report the value implied by the structural estimates computed from [Table 4](#) that corresponds to the OLS coefficient of the subsequent columns. Column [1] shows that the relationship between productivity and

the share of senior executives is positive for conglomerate controlled firms: the coefficient is 0.11 with a standard error of 0.040. To give a sense of the size of the effect, increasing the share of executives with more than 5 years of tenure by one standard deviation (0.30, [Table 2](#)) would increase productivity by around 3%. The value is comparable with that of the structural estimates (0.18). The selection effect is around 3% weaker in foreign controlled firms – exactly the same difference implied by the structural estimates — and the estimate are not statistically different from zero. In family firms, senior executives are on average 17% less productive than in the conglomerate controlled firms. Although larger in absolute value than the value implied by the structural estimate – 12% with a standard error of 0.05 we cannot reject the hypothesis that the two values are the same at conventional levels of significance. Finally, we obtain a very large negative coefficient for government firms (-0.47). This value implies a negative selection in such firms, i.e., that junior executives are more efficient than senior ones. This is an outcome that, by construction, our structural model cannot predict, confirming that government firms display some features that the model cannot match, as discussed in the previous section. However, the sign of the effect is consistent with the structural results: selection is at best absent in government controlled firms.

We have performed a series of robustness checks. First, given that family firms are on average smaller than the rest of the sample, in Column [2] we include firm size (log employment) finding no significant difference. In column [3] we do not weight observations. In this case, the results are somehow weaker: the coefficient on  $\phi$  is positive but statistically insignificant. The interaction terms for family and government are negative and significant, again pointing to weaker selection in these firms. Finally, in Column [4] we use a 7-year based definition of seniority and in column [5] a 3-year definition, with again no substantial differences with respect to the basic specification.

We have performed a final set of robustness checks with respect to the measure of performance. First, we have directly run some production function estimates, rather than using the two step procedure that first estimates TFP and then relates it to the share of senior

Table 6: TFP and share of senior executive relationship, by Control type

	Dependent variable: log TFP					
	Structural	[1]	[2]	[3]	[4]	[5]
$\phi$	0.10	0.111*** (0.040)	0.115*** (0.039)	0.022 (0.034)	0.081* (0.044)	0.089** (0.040)
$\phi \cdot \text{Foreign}$	-0.03	-0.035 (0.063)	-0.040 (0.064)	0.012 (0.049)	0.020 (0.069)	-0.053 (0.063)
$\phi \cdot \text{Family}$	-0.12	-0.175*** (0.047)	-0.178*** (0.047)	-0.096** (0.040)	-0.170*** (0.052)	-0.127** (0.049)
$\phi \cdot \text{Government}$	-0.18	-0.469*** (0.088)	-0.473*** (0.088)	-0.316*** (0.074)	-0.417*** (0.105)	-0.370*** (0.089)
log labour			0.007 (0.007)			
Observations		5,875	5,875	5,943	4,897	6,840
$R^2$		0.47	0.47	0.49	0.45	0.47

Note: The Column “Structural” reports the statistics corresponding to the structural estimates, derived from [Table 4](#).  $\phi$  is the share of senior executives who have been with the firm at least 5 years in columns [1]-[3], at least 7 years in column [4] and at least 3 years in column [5]. All regressions are weighted with sampling weights with the exception of column [3], which is unweighted. All regressions include control type dummies, year dummies, 2-digit sector dummies. Robust standard errors in parenthesis.

executives. Second, we have used a profit-based measure of performance. In fact, one might argue that profits is the correct indicator of managerial ability, and that efficiency and profitability might not be simply one-to-one ([Foster, Haltiwanger, and Syverson, 2008](#)). We have therefore used returns on assets as the dependent variable. The results of both exercises are reported and discussed in [Appendix B](#). They are aligned with those of [Table 6](#).

To sum up, the model-based OLS regressions are fully consistent with the structural estimates, although based on completely different dimensions of data variability. They also suggest that the structural estimates are robust with respect to the values of the auxiliary parameters, that do not enter the OLS estimates.

## 8 Concluding remarks

We formulated a model of executive selection in which the firm's owner cares about managerial ability and, in addition, derives a private benefit from developing a personal relationship with the executives. The model predicts that, compared to an owner who is only interested in ability, the managers' selection in the case of multiple objectives becomes less effective and reduces the firm's productivity.

The theory yields joint predictions on two observables: the firm's average productivity and the share of senior executives in the firm. These predictions can be inverted to infer the structural parameters of the model, in particular to learn how important is the personal-relationship value of executives enjoyed by the firm's owner. A structural estimation of the model, based on matched employer-employee data in a sample of Italian manufacturing firms, shows that the non-monetary objectives appear quantitatively important in accounting for the data. In particular, the value of personal-relationship is highest in the firms under government-control, and smallest in conglomerate or foreign owner firms. From a quantitative point of view, those differences account for a 10% differential in the firms' total factor productivity. These results are robust to several controls and estimation methods. We also show that the structural estimates are fully consistent with a large number of model-based OLS regressions.

Our results suggest that, in countries dominated by either family or government controlled firms, the process of executive selection leaves unexploited efficiency gains on the table, because incompetent executives are appointed. These results square with anecdotal evidence on the Italian case, where a fairly restricted pool of executives run the largest companies, often with careers unaffected by bad performance. Of course, we cannot draw welfare conclusions, as the private benefits do deliver utility to owners. Still, the paper points to one possibly important drawback of family and government firms. One important question is then what mechanisms could mitigate the inefficiency in executive selection. We expect that competition in the product markets and contestability of control would reduce the extent of

such inefficiency. We plan to explore this in future work.

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# A Proofs

This appendix provides the proofs of the three propositions in the paper. The arguments are based on standard analysis and probability notions.

## A.1 Proof of Proposition 1.

**Proof.** Simple algebra shows that  $H(s^*, R)$ : is continuous in  $s^*$ , that  $H(0, R) < 0$ , and that the first order derivative w.r.t.  $s^*$  is positive,  $H_{s^*}(s^*, R) = 1 + \beta(\rho - F(s^*)) > 0$ , and in the limit  $\lim_{s^* \rightarrow \infty} H_{s^*}(s^*, R) > 0$ . Hence there exist one and only one  $s^* > 0$  that solves equation (9).

We now show that the implicit function  $s^*(R)$  is increasing in  $R$ . Applying the implicit function theorem to equation (9) gives

$$\frac{\partial s^*}{\partial R} = \frac{(1 - \beta) \frac{\partial v_y}{\partial R}}{1 - (1 - \beta) \frac{\partial v_y}{\partial s^*}} \quad (\text{A-1})$$

Let us use expression (7) to compute:

$$\begin{aligned} \frac{\partial v_y}{\partial s^*} &= \frac{-\beta s^* f(s^*) [(1 - \beta)(1 + \beta(\rho - F(s^*))) + \beta(1 - \beta)f(s^*) [\mu(1 - \beta(1 - \rho)) + \beta \int_{s^*}^{\infty} s dF(s)]]}{[(1 - \beta)(1 + \beta(\rho - F(s^*)))]^2} \\ &= \beta f(s^*) \frac{-s^* + (1 - \beta) \frac{\mu(1 - \beta(1 - \rho)) + \beta \int_{s^*}^{\infty} s dF(s)}{(1 - \beta)(1 + \beta(1 - F(s^*)))}}{(1 - \beta)(1 + \beta(1 - F(s^*)))} = \beta f(s^*) \frac{-s^* + (1 - \beta)v_y}{(1 - \beta)(1 + \beta(1 - F(s^*)))} . \end{aligned}$$

Using that at the optimum  $(1 - \beta)v_y = s^*$  gives  $\frac{\partial v_y}{\partial s^*} = 0$ . Hence  $\frac{\partial s^*}{\partial R} = (1 - \beta) \frac{\partial v_y}{\partial R}$ . Next, we show that  $0 < (1 - \beta) \frac{\partial v_y}{\partial R} < 1$ . Rewrite the integral term in the numerator of (7) as

$$\begin{aligned} \int_{s^*}^{\infty} s dF(s) &= q \int_{s^* - R}^{\infty} (x + R) dG(x) + (1 - q) \int_{s^*}^{\infty} x dG(x) \\ &= q \left( R(1 - G(s^* - R)) + \int_{s^* - R}^{\infty} x dG(x) \right) + (1 - q) \int_{s^*}^{\infty} x dG(x) \end{aligned}$$

Using this expression in (7) and taking the derivative w.r.t.  $R$  yields:

$$\begin{aligned} (1 - \beta) \frac{\partial v_y}{\partial R} &= \beta \frac{\frac{\partial \int_{s^*}^{\infty} s dF(s)}{\partial R} + (1 - \beta) v_y \frac{\partial F(s^*)}{\partial R}}{1 + \beta(1 - F(s^*))} \\ &= \beta q \frac{[1 - G(s^* - R) + Rg(s^* - R) + (s^* - R)g(s^* - R)] - s^*g(s^* - R)}{1 + \beta(1 - F(s^*))} \\ &= \beta q \frac{1 - G(s^* - R)}{1 + \beta(1 - F(s^*))} \in (0, 1) \end{aligned}$$

where the second equality uses  $s^* = (1 - \beta)v_y$ .

Finally we show that  $s^* > \mu$ . Note that for  $R = 0$  equation (1) gives  $F(z) = G(z)$ . Using



equation (9) to evaluate and  $H(s^*, R)$  at  $R = 0$  gives

$$H(s^*, 0) = (s^* - \mu)(1 + \beta \rho) + \beta \left( \mu - \int_{s^*}^{\infty} z dG(z) - s^* G(s^*) \right)$$

Simple algebra shows that at  $s^* = \mu$  we have that  $H(\mu, 0) < 0$  (since  $\mu - \int_{\mu}^{\infty} z dG(z) < 0$ ). Using that  $H(s^*, R)$  is increasing in  $s^*$  implies that  $s^*(0) > \mu$ . Using that  $s^*(R)$  is increasing in  $R$ , implies that  $s^*(R) > \mu$  for any  $R \geq 0$ . ■

## A.2 Proof of Proposition 2.

**Proof.** Rewrite the average productivity  $X$  defined in equation (13) as

$$X = \mu + \frac{q \int_{s^*-R}^{\infty} x dG(x) + (1-q) \int_{s^*}^{\infty} x dG(x) - \mu(1 - F(s^*))}{1 + \rho - F(s^*)}$$

The parameter  $R$  enters this expression directly and via  $s^*$ . Taking the first order derivative with respect to  $R$ , accounting for both direct and indirect effects, gives (after some algebra and collecting terms):

$$\begin{aligned} \frac{\partial X}{\partial R} = & \frac{1}{1 + \rho - F(s^*)} \cdot \left[ q R g(s^* - R) \frac{\partial(s^* - R)}{\partial R} + \right. \\ & \left. \frac{\partial F(s^*)}{\partial R} \left( \frac{\rho \mu + q \int_{s^*-R}^{\infty} x dG(x) + (1-q) \int_{s^*}^{\infty} x dG(x) - \mu(1 - F(s^*))}{1 + \rho - F(s^*)} - s^* \right) \right] \end{aligned} \quad (\text{A-2})$$

Now use equation (9) with  $\beta = 1$  to get the following implicit equation for  $s^*$

$$s^* = \frac{\rho \mu + \int_{s^*}^{\infty} s dF(s)}{1 + \rho - F(s^*)} = \frac{\rho \mu + q \int_{s^*-R}^{\infty} x dG(x) + (1-q) \int_{s^*}^{\infty} x dG(x) + q R (1 - G(s^* - R))}{1 + \rho - F(s^*)}$$

Replacing the expression on the right hand side for  $s^*$  into equation (A-2) and using the expression for  $\frac{\partial F(s^*)}{\partial R}$  computed in equation (12) gives (after some rearranging and cancellations)

$$\begin{aligned} \frac{\partial X}{\partial R} = & \frac{q R}{1 + \rho - F(s^*)} \left[ g(s^* - R) \left( \frac{1 + \rho - (q + (1-q)G(s^*))}{1 + \rho - F(s^*)} \right) \frac{\partial(s^* - R)}{\partial R} \right. \\ & \left. - \frac{1 - G(s^* - R)}{1 + \rho - F(s^*)} (1-q)g(s^*) \frac{\partial s^*}{\partial R} \right] \end{aligned} \quad (\text{A-3})$$

Inspection of equation (A-3), and the results on the sign of the partial derivatives established in Proposition 1, reveals that the derivative is zero at  $R = 0$ , and that it is negative at  $R > 0$ . ■

### A.3 Proof of Proposition 3.

**Proof.** Define  $\zeta_{i,j}$  as the deviation of a senior executive  $j$  from the productivity of senior executives,  $X_o$ . Analogously, let  $\xi_{i,j}$  be the deviation of a junior executive  $j$  from  $\mu$ . Naturally the expected value of those deviations is zero. In a small sample of size  $n$ , the average productivity of junior and senior incumbent executives in firm  $i$  can be written as:

$$X_{o,i} = X_o + \frac{\sum_{j=1}^{n_{o,i}} \zeta_{i,j}}{n_{o,i}} \quad , \quad X_{y,i} = \mu + \frac{\sum_{j=1}^{n-n_{o,i}} \xi_{i,j}}{n-n_{o,i}}$$

$$\text{Then, } \varepsilon_i \equiv \phi_i \left( \frac{\sum_{j=1}^{n_{o,i}} \zeta_{i,j}}{n_{o,i}} - \frac{\sum_{j=1}^{n-n_{o,i}} \xi_{i,j}}{n-n_{o,i}} \right) + \frac{\sum_{j=1}^{n-n_{o,i}} \xi_{i,j}}{n-n_{o,i}} \quad .$$

We show that  $\text{cov}(\phi_i, \varepsilon_i) = 0$ , so that the OLS regression assumptions are satisfied. Let  $n$  be the number of executives in each firm. For notational convenience let us define  $z_i \equiv \frac{\sum_{j=1}^{n_{o,i}} \zeta_{i,j}}{n_{o,i}}$  and  $u_i \equiv \frac{\sum_{j=1}^{n-n_{o,i}} \xi_{i,j}}{n-n_{o,i}}$ , to write:

$$\text{cov}(\phi_i, \varepsilon_i) = \mathbb{E}[\phi_i^2 (z_i - u_i) + \phi_i u_i] - \mathbb{E}(\phi_i) \mathbb{E}[\phi_i (z_i - u_i) + u_i]$$

The key of the proof is that the conditional expectation  $\mathbb{E}(z_i | n_{o,i} = k) = 0$ , for all  $k = 0, 1, \dots, n$ . To see this note that, for a given  $k$ :

$$\mathbb{E} \left[ \frac{\sum_{j=1}^k \zeta_{i,j}}{k} \mid n_{o,i} = k \right] = \frac{1}{k} \sum_{j=1}^k \mathbb{E}(\zeta_{i,j} | x_{i,j} + r_{i,j} > s^*) = 0$$

This holds since  $\mathbb{E}(\zeta_{i,j} | x_{i,j} + r_{i,j} > s^*) = 0$  for each  $j$ . Recall that  $\zeta_{i,j}$  is the deviation of a senior executive productivity  $x_j$  from the senior executive unconditional productivity,  $X_o$ . It is immediate that conditioning on the information that an executive is tenured does not provide any information on how much above (or below) the average tenured executives' level ( $X_o$ ) he is.

Recall that  $\phi_i$  takes the values  $(0, \frac{1}{n}, \dots, \frac{k}{n}, \dots, 1)$ . As productivity realization are independent across executives, the probability of each  $\phi_i = \frac{k}{n}$  outcome is  $\Pr\left(\frac{k}{n}\right) \equiv p(k, n)$  from a binomial distribution. Then (for  $a = 1, 2$ )

$$\begin{aligned} \mathbb{E}_{\phi,z}(\phi_i^a z_i) &= \mathbb{E}_{\phi}[\mathbb{E}_z(\phi^a z_i) | \phi_i = \phi] = \sum_{k=0}^n p(k, n) \mathbb{E}_z \left[ \left(\frac{k}{n}\right)^a \cdot z_i | n_{o,i} = k \right] \\ &= \sum_{k=0}^n p(k, n) \left(\frac{k}{n}\right)^a \mathbb{E}_z[z_i | n_{o,i} = k] = \sum_{k=0}^n p(k, n) \left(\frac{k}{n}\right)^a \cdot 0 = 0 \end{aligned}$$

The same logic shows that  $\mathbb{E}_{\phi,u}(u_i \phi_i^a) = 0$  for  $a = 1, 2$ . This is immediate as the productivity of the junior is not observed by the principal, hence it cannot be correlated with his decisions about the tenure of the senior executives. ■

## B Data and OLS regressions details

The INVIND survey is based on a questionnaire comprised of a fixed and monographic sections that change from year to year, used to investigate in-depth aspects of firms' activity. In 1992 a large section was devoted to corporate control. The determination of the nature of the controlling shareholders begins with that year. Among other things, the questionnaire asked about each firm's main shareholder, distinguishing between 10 different categories. Since 1992, the questions on control structure have been included every year. Starting in 1996, the categories have been reduced to five: 1) individual or family; 2) government (local or central or other publicly controlled entities); 3) conglomerate; 4) institution (financial or not); 5) foreign owner. We collapse the last two categories into one and map the previous classification into these four groups. Before 1992 the nature of the controlling shareholder was not investigated. However, in 1992 the firm was asked the year of the most recent change in control. We extend the control variable of 1992 back to the year of the most recent control change. Moreover, if a firm has a certain controller type in year  $t$  and the same in year  $t'$ , and some missing values in the year in between, we assume that the control has remained of the same type for all the period  $t - t'$ . Note that there might be some cases of misclassification, in particular among firms that are classified as not controlled by an individual. For example, a foreign entity controlling a resident firm might in turn be controlled by a resident that uses the offshore firm for taxation purposes. The same holds true for firms that report an institution as the controlling shareholder. This would bias the difference in the estimates between family and non family firms downward, because we would be classifying as foreign some family firms (the opposite case is not very likely). This implies that our results can be seen as a lower bound of the difference we find.

The CADS data are used to construct the capital stock using the permanent inventory method. Investment is at book value, adjusted using the appropriate two-digit deflators and depreciation rates, derived from National Accounts published by the National Institute for Statistics. For consistency with the capital data, in the estimation of the production function we take value added and labor from the CADS database. Both the INVIND and the CADS samples are unbalanced, so that not all firms are present in all years.

Data on workers are extensively described in [Iranzo, Schivardi, and Tosetti \(2008\)](#). We cleaned the data by eliminating the records with missing entries on either the firm or the worker identifier, those corresponding to workers younger than 25 (just 171 observations, .08% of the total) and those who had worked less than 4 weeks in a year. We also avoided duplication of workers within the same year; when a worker changed employer, we considered only the job at which he had worked the longest.

The main econometric problem in recovering TFP is that inputs are a choice variable and thus are likely to be correlated with unobservables, particularly the productivity shock. This is the classical problem of endogeneity in the estimation of production functions. To deal with it we follow the procedure proposed by [Olley and Pakes \(1996\)](#). Using a standard dynamic programming approach, Olley and Pakes show that the unobservable productivity shock can be approximated by a non-parametric function of the investment and the capital stock. To allow for sectoral heterogeneity in the production function, we estimate it separately at the sectoral level. The estimation procedure, the coefficients, and all the results are described in details in [Cingano and Schivardi \(2004\)](#).

To make sure that our results are not dependent on the TFP measure, we also perform some direct production function estimation exercises. To control for endogeneity we again follow Olley and Pakes and include in the regression a third degree polynomial series in  $i$  and  $k$  and their interactions, which approximate the unobserved productivity shock.<sup>18</sup> In [Table A-1](#) we report a series of exercises analogous to those of [Table 6](#) in the main text. The dependent variable is log value added, the regressors are capital and labor in addition to the share of senior executives interacted with the control dummies, and the control dummies themselves. All the regressions include year and sectoral dummies. As in [Table 6](#), we use sample weights with the exception of column [3]. In column [1] we do simple OLS; the Olley and Pakes controls are introduced in Column [2] and maintained throughout; in Column [3] we do not weight observations. Column [4] uses a 7-year period to become senior and column 5 a 3-year period. Results are similar across specifications; more importantly, they are very much in line with those of [Table 6](#).

One potential objection to our exercise is the performance measure. One can in fact argue that owners select executives in terms of their contribution to firm profits, rather than productivity. We have argued that profits are an increasing function of productivity, and that TFP measures are more precise than accounting measures of profits. However, profitability and productivity are not necessarily one-to-one, as the recent work of [Foster, Haltiwanger, and Syverson \(2008\)](#) show. In the OLS framework we can directly test if the patterns that emerge for productivity also hold for profitability measures. In [Table A-2](#) we use return on assets (ROA) as dependent variable. In this case, the coefficient on the share of senior executives can be interpreted as the difference in the average contribution to ROA of senior and junior executives. The patterns we find are exactly the same as those emerging with the productivity measures, if anything stronger. In particular, profitability is positively related to the share of senior executives in conglomerate controlled firms. To give a sense of the size of the effect, increasing the share of executives with more than five years of tenure by one standard deviation (0.3, [Table 2](#)) would increase ROA by 1 percentage point (the median ROA is 7.8, the mean 8.6). The interaction for foreign control firms is not significantly different from zero, while it is negative and significant for family and government firms. Again, for the latter the effect is very strong, indicating negative selection in such firms. This is fully consistent with TFP results. In particular, in family and government firms the selection effect for senior executives is absent compared to the other control types. Similar results are obtained when using return on equity. This shows that our results are robust with respect to different performance measures.

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<sup>18</sup>Note that when the nonparametric term in capital and investment is included, the capital coefficient can no longer be interpreted as the parameter of the production function in the first stage of the procedure. However, given that the coefficient on capital is of no particular interest to us, this is inconsequential for our purposes.

Table A-1: Value added and share of senior executives, by control type

	<b>Dependent variable: log value added</b>				
	[1]	[2]	[3]	[4]	[5]
$\phi$	0.131*** (0.039)	0.127*** (0.039)	0.056* (0.032)	0.102** (0.043)	0.109*** (0.039)
$\phi \cdot \text{Foreign}$	-0.088 (0.061)	-0.060 (0.060)	-0.030 (0.046)	0.003 (0.066)	-0.072 (0.061)
$\phi \cdot \text{Family}$	-0.204*** (0.046)	-0.186*** (0.045)	-0.115*** (0.038)	-0.179*** (0.050)	-0.137*** (0.046)
$\phi \cdot \text{Government}$	-0.471*** (0.084)	-0.442*** (0.081)	-0.304*** (0.072)	-0.370*** (0.098)	-0.347*** (0.085)
log labor	0.762*** (0.012)	0.715*** (0.012)	0.745*** (0.009)	0.713*** (0.013)	0.702*** (0.012)
log capital	0.241*** (0.011)				
Observations	5,875	5,811	5,828	4,841	6,756
$R^2$	0.86	0.87	0.92	0.87	0.87

Note:  $\phi$  is the share of senior executives who have been with the firm at least 5 years in columns [1]-[3], at least 7 years in column [4] and at least 3 years in column [5]. All regressions are weighted with sampling weights with the exception of column [3], which is unweighted. Following [Olley and Pakes \(1996\)](#), regressions in columns 2-5 include a 3rd degree polynomial in capital and investment to control for the unobserved productivity shock. All regressions include control type dummies, year dummies, 2-digit sector dummies. Robust standard errors in parenthesis.

Table A-2: ROA and share of senior executives, by control type

	Dependent variable: ROA				
	[1]	[2]	[3]	[4]	[5]
$\phi$	3.40*** (0.75)	3.44*** (0.75)	2.93*** (0.66)	2.30*** (0.85)	3.22*** (0.74)
$\phi \cdot \text{Foreign}$	-1.57 (1.38)	-1.62 (1.40)	0.24 (1.07)	-0.92 (1.35)	-0.71 (1.36)
$\phi \cdot \text{Family}$	-4.48*** (0.95)	-4.50*** (0.95)	-3.69*** (0.78)	-3.32*** (1.07)	-4.25*** (0.96)
$\phi \cdot \text{Government}$	-7.50*** (1.61)	-7.54*** (1.61)	-3.73** (1.50)	-4.71** (1.84)	-7.22*** (1.67)
1		0.07 (0.17)			
Observations	5,875	5,875	5,943	4,897	6,840
$R^2$	0.08	0.08	0.09	0.08	0.08

Note: ROA is return on assets, in percentage units.  $\phi$  is the share of senior executives who have been with the firm at least 5 years in columns [1]-[3], at least 7 years in column [4] and at least 3 years in column [5]. All regressions are weighted with sampling weights with the exception of column [3], which is unweighted. All regressions include control type dummies, year dummies, 2-digit sector dummies. Robust standard errors in parenthesis.

## C Derivation of the likelihood function

The likelihood for a sample of observations  $Y$ , under the parametrization  $\Theta$  is

$$\begin{aligned} L(\Theta; Y) &= \prod_{\kappa=1}^K \prod_{j=1}^2 \prod_{i=1}^{n_\kappa} \frac{1}{(2\pi\sigma_j^2)^{1/2}} \exp\left(-\frac{1}{2} \left[ \frac{y_{i,\kappa}^j - f^j(\Theta, \kappa)}{\sigma_j} \right]^2\right) \\ &= \prod_{\kappa=1}^K \prod_{j=1}^2 (2\pi\sigma_j^2)^{-n_\kappa/2} \prod_{i=1}^{n_\kappa} \exp\left(-\frac{1}{2} \left[ \frac{y_{i,\kappa}^j - f^j(\Theta, \kappa)}{\sigma_j} \right]^2\right) \end{aligned}$$

where  $K$  is the number of “groups” in the model (4 control types in our case). This is

$$\log L(\Theta; Y) = -\frac{1}{2} \sum_{\kappa=1}^K \sum_{j=1}^2 n_\kappa \log(2\pi\sigma_j^2) - \frac{1}{2} \sum_{\kappa=1}^K \sum_{j=1}^2 \sum_{i=1}^{n_\kappa} \left[ \frac{y_{i,\kappa}^j - f^j(\Theta, \kappa)}{\sigma_j} \right]^2 \quad (\text{A-4})$$

For all observable  $j$  and for each group  $\kappa$  (hence omitting the  $j, \kappa$  subindices)

$$\begin{aligned} \sum_{i=1}^{n_\kappa} \left[ \frac{y_i - f}{\sigma} \right]^2 &= \frac{n_\kappa}{\sigma^2} \left[ \frac{\sum_{i=1}^{n_\kappa} y_i^2}{n_\kappa} + f^2 - 2 \frac{\sum_{i=1}^{n_\kappa} y_i f}{n_\kappa} \right] \\ &= \frac{n_\kappa}{\sigma^2} \left[ \sigma^2 + \left( \frac{\sum_{i=1}^{n_\kappa} y_i}{n_\kappa} \right)^2 + f^2 - 2 \frac{\sum_{i=1}^{n_\kappa} y_i f}{n_\kappa} \right] \\ &= n_\kappa + \frac{n_\kappa}{\sigma^2} (f - \bar{y}_\kappa)^2 \quad \text{where } \bar{y}_\kappa \equiv \frac{\sum_{i=1}^{n_\kappa} y_i}{n_\kappa} \end{aligned}$$

Replacing this expression in [equation \(A-4\)](#) we can rewrite the likelihood function by minimizing the distance between the theoretical value  $f(\Theta, \kappa)$  and the sample average  $\bar{y}_\kappa^j$  for each variable  $j$ , or

$$\log L(\Theta; Y) = -\frac{1}{2} \sum_{\kappa=1}^K \sum_{j=1}^2 n_\kappa \ln(2\pi\sigma_j^2) - \frac{1}{2} \sum_{\kappa=1}^K \sum_{j=1}^2 \left[ n_\kappa^j + \frac{n_\kappa^j}{\sigma_j^2} (f^j(\Theta, \kappa) - \bar{y}_\kappa^j)^2 \right] \quad (\text{A-5})$$

The measurement error for variable  $j$  (common for all group  $\kappa$ ) is

$$\sigma_j^2 \equiv \text{var}(y^j) = \sum_{i=1}^{n_\kappa} \frac{1}{n_\kappa} (y_{i,\kappa}^j - \bar{y}_\kappa^j)^2$$

## C.1 Score and Information matrix

Let  $M$  be the size of  $\Theta$ . The  $n$ -th element of the score is given by

$$\begin{aligned} \mathbf{s}_n(\Theta; Y) &\equiv \frac{\partial \log L(\Theta; Y)}{\partial \theta_n} = -\frac{1}{2} \frac{\partial F(\Theta; Y)}{\partial \theta_n} \\ &= \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) (\bar{y}_\kappa^j - f^j(\Theta, \kappa)) \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \end{aligned}$$

The  $(n, m)$  element of the  $M \times M$  information matrix  $I(\Theta)$  is defined as:

$$I_{n,m}(\Theta) = \mathbb{E} \left[ \frac{\partial \log L(\Theta; Y)}{\partial \theta_n} \frac{\partial \log L(\Theta; Y)}{\partial \theta_m} \right] = \mathbb{E} [\mathbf{s}_n(\Theta, Y) \mathbf{s}_m(\Theta, Y)]$$

which in our case becomes

$$\begin{aligned} I_{n,m}(\Theta) &= \mathbb{E} \left\{ \left[ \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) (\bar{y}_\kappa^j - f^j(\Theta, \kappa)) \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \right] \right. \\ &\quad \left. \left[ \sum_{\kappa'=1}^K \sum_{j'=1}^2 \left( \frac{n_{\kappa'}}{\sigma_{j'}^2} \right) (\bar{y}_{\kappa'}^{j'} - f^{j'}(\Theta, \kappa')) \frac{\partial f^{j'}(\Theta, \kappa')}{\partial \theta_m} \right] \right\} \\ &= \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) \sum_{\kappa'=1}^K \sum_{j'=1}^2 \left( \frac{n_{\kappa'}}{\sigma_{j'}^2} \right) \\ &\quad \mathbb{E} \left\{ (\bar{y}_\kappa^j - f^j(\Theta, \kappa)) (\bar{y}_{\kappa'}^{j'} - f^{j'}(\Theta, \kappa')) \right\} \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \frac{\partial f^{j'}(\Theta, \kappa')}{\partial \theta_m} \\ &= \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) \sum_{\kappa'=1}^K \sum_{j'=1}^2 \left( \frac{n_{\kappa'}}{\sigma_{j'}^2} \right) \\ &\quad \mathbb{E} \left\{ \left( \frac{1}{n_\kappa} \sum_{i=1}^{n_\kappa} \varepsilon_i^j \right) \left( \frac{1}{n_{\kappa'}} \sum_{i=1}^{n_{\kappa'}} \varepsilon_i^{j'} \right) \right\} \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \frac{\partial f^{j'}(\Theta, \kappa')}{\partial \theta_m} \\ &= \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) \left( \frac{n_\kappa}{\sigma_j^2} \right) \left\{ \frac{\sigma_j^2}{n_\kappa} \right\} \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \frac{\partial f^{j'}(\Theta, \kappa')}{\partial \theta_m} \\ &= \sum_{\kappa=1}^K \sum_{j=1}^2 \left( \frac{n_\kappa}{\sigma_j^2} \right) \frac{\partial f^j(\Theta, \kappa)}{\partial \theta_n} \frac{\partial f^{j'}(\Theta, \kappa')}{\partial \theta_m} \end{aligned}$$