

# Liability Rules and Optimal Care for Large Accidents

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## Abstract

This paper revisits the model of the unilateral accident, introducing three assumptions which depart from the literature: parties are Rank Dependant Expected Utility maximizers, allowing us to capture two important behavioral attitudes towards risk, i.e. pessimism (probability transformation) and risk aversion; there exists an aggregate/uninsurable risk in case of accident (rather than individual risks); tortfeasors have the opportunity to invest in damages reduction activities (rather than, in probability reduction expenditures). We first study the properties of efficient care and efficient risk sharing. We show that the optimal level of care is larger than under the risk neutral/small risks case, and that it depends on the aggregate wealth of society but not on wealth distribution across parties. We also study the effect of pessimism on care expenditures and show that it is ambiguous in contrast to the influence of risk aversion. Finally, we show that ordinary liability rules are no longer equivalent, generally inefficient, and that negligence does not dominate strict liability.

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# 1 Introduction

In the literature on *Tort Law*, the economic model most widely used (see Shavell (1987)) is a very stylized situation of a unilateral accident: one party (one or several victims) is injured by an accident due to the wrongdoing of a second party (an injurer). The accidental event which yields in the harm/damage of the victim(s) is an avoidable by-product of the main productive activity of the injurer, which is otherwise socially useful. The injurer has the opportunity to invest in a safety technology (he may undertake precautions, or care, denoted  $x$ ) to reduce the level of expected damage when the accident occurs (denoted  $ph(x)$ , with  $p$  the probability of accident, and  $h(x)$  the damage/loss of the victim). Both individuals are supposed to be risk-neutral: this basic framework will be labelled a *risk-neutral world*. In this case, any feasible allocation of risk (any allocation of the aggregate wealth) is Pareto efficient, and the first-best level of care satisfies (see Calabresi (1970), Diamond (1974a,b), Green (1976)):

$$-ph'(\hat{x}) = 1 \tag{1}$$

meaning that the *risk-neutral or risk-free level of care*  $\hat{x}$  is set such that the planner weighs the victim's expected benefit resulting from the loss reduction (LHS of the equality) and the marginal cost of care borne by the injurer (RHS). Remark that Shavell (1982) obtains the same result for the level of care but in an economy with *purely idiosyncratic* risks: when there is no aggregate risk to be shared (*i.e.* soon as the expected aggregate wealth is constant across the states of the world), the issue of the efficient level of safety becomes independent from the issue of the allocation of risks. It is easy to verify that  $\hat{x}$  increases in  $p$  but depends neither on society's aggregate wealth, nor on the distribution of such wealth among individuals – it only depends on the characteristics of the technology of prevention available to this economy.

In other words, this standard risk-neutral model relies on situations where parties (the injurer as well as the victim(s)) have the same knowledge and assessment of the risk of accident, and where this knowledge is consistent with a Bayesian representation (Expected Utility representation of preferences). On the other hand, this framework fits well situations where specific institutions specialized in the management of risks are available, while safety activity is decentralized to private entities; then a complete separation is obtained between the allocation of risks in the society (among private entities or individuals), the compensation of losses to the victims of accident, and the design of incentives to invest in safety. This requires that small (individual) risks only exist in the economy and are (fully) insurable.

In contexts where exist numerous victims and where risk aversion matters<sup>1</sup>, strict liability is widely seen as the most suitable way to govern highly risky activities (environmentally or healthy dangerous production). The argument is that strict liability is supposed to induce both efficient care and an efficient level of the risky activity itself, whereas negligence will lead to a higher (inefficient)

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<sup>1</sup>Shavell (1982) discusses the role of risk aversion in presence of moral hazard.

level of output. Nell and Richter (2003) discuss this point, considering a world of Expected Utility parties with constant absolute risk aversion. They show that, if insurance markets are imperfect, the negligence rule implies a result in terms of risk sharing that should be preferred. The reason is that since highly risky activities typically affect a large number of individuals, then strict liability implies a quite unfavorable allocation of risk. Therefore, the negligence rule turns out to be superior, if a market relationship between the parties exists, since it incurs less cost of risk. If there is no market relationship between injurer and victims, no clear result can be derived. Graff Zivin and ali (2006) investigates the performance of liability rules in a sequential model of the bilateral accident case. They find that an increase in injurer liability does not necessarily increase safety or efficiency in cases where the injurer is risk neutral. Complete injurer liability is found to yield Pareto optimality. When either party is risk averse, an increase in injurer liability may sometimes reduce safety and efficiency. If the injurer is risk neutral and the victim is risk averse, Pareto optimality is only achieved by assigning full liability on the injurer. If the injurer is risk averse and the victim is risk neutral, no level of injurer liability is optimal. In this case, optimality can only be achieved through the contractualization of abatement activities.

Indeed, most accidents on industrial plants (chemical, nuclear and so on ...) are large ones, in the sense that they injure a large number of victims at the same time: typically, these are catastrophic losses/small probability events<sup>2</sup>. On the one hand, they are for a main part not insurable, since the size and concentration of the total loss implies a large risk of bankruptcy for the insurers<sup>3</sup>. On the other hand, in these contexts of small probability events, there exist several bias in individuals' risk perception (probability distortions, certainty effects and so on).

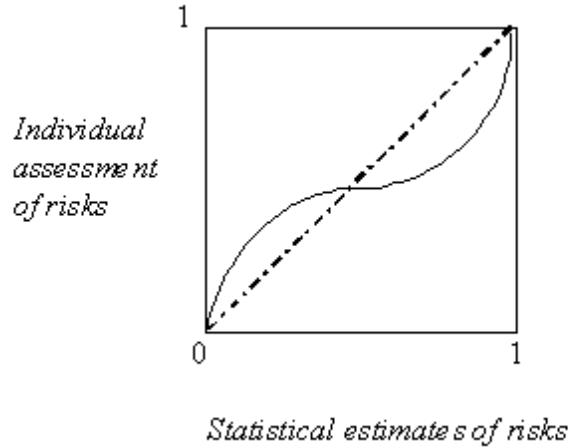
What is observed for a great majority of people in various context is a tendency to under estimate events associated with a large probability of occurrence, but at the same time to over assess events with a small probability. For example, the typical patterns of behaviors/responses observed in experimental works suggest a inverse *U-shaped* probability transformation function as depicted in graph 1 (Tversky and Wakker (1995), Abdellaoui (2000), Stott (2006)): individuals undertake risk seeking decisions when they face low probabilities of winning or large probabilities of loosing (the transformation is concave), and simultaneously, they follow risk averse behavior when they face small probabilities of losses or large probabilities of gains (the transformation is convex). Such a probability transformation process is also observed when people are asked to assess the frequency of fatal hazards to which they may be faced during their lifetime (see Lichtenstein and ali, 1978).

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<sup>2</sup>Usual analysis of liability rules for environmental dammages do not consider this issue (see Cooter (1986), Hansen and Thomas (1999), Fees and ali (2009), Watabe (1999)). The paper by Segerson (1986) is close to the point we made here regarding the existence of a large risk, but does not go on into details. Most recently, a literature emerges regarging liability and innovation and/or technological risks, see for example Dari-Mattiacci and Franzoni (2010), Endres and Bertram (2006), Endres and Friehe (2011a,b) and Jacob (2010).

<sup>3</sup>The role of financial markets (catastrophic bonds) in the sharing of large risks is beyond the scope of this paper.

GRAPH 1 : A typical probability distortion function



Regarding firms and/or managers decisions, Adellaoui and Munier (1997) have also shown that this specific pattern of behavior is observed in the area of industrial activities associated with potential major accidents: employees including highly graduated or skilled ones (engineers, technicians) of chemical or nuclear plants exhibited a tendency to underestimate the probability of accident, leading them to undertake lower efforts of precaution than needed. Moreover, it is now well documented that both the demand and supply sides of insurance markets display these bias (Hogarth and Kunreuther (1985, 1989), Kunreuther and ali (1993, 1995)) – which induces severe market insurance failures. Insurance coverage may be not available but for high premia, and indemnity schedules are associated with fixed reimbursements, including large deductibles on small losses and caps or upper limits on large ones; thus their exist at best only limited opportunities of insurance coverage.

Thus, the existence of bias in risk perception associated with imperfect insurance markets, becomes a main concern for the control of risky activities through tort law. Several authors have tried to tackle the issue of risk perception in the analysis of Tort Law<sup>4</sup>.

Bigus (2006) discusses the functioning of tort law, when tortfeasors have preferences satisfying the axioms of the *Prospect Theory*. He finds, to the extend that high probabilities are under-estimated while low ones are over-estimated, that the level of care obtained under strict liability is too low as compared to the efficient one; in contrast, negligence may reach the first best. Eide (2007) finds similar results when offenders have preferences corresponding to the *Rank Dependent Expected Utility Theory*. That *Prospect Theory* and *Rank Dependent*

<sup>4</sup>Dari-Mattiacci (2005) and de Geest and Dari-Mattiacci (2005) consider the different issue of Court's bias in the determination of liability or dammage.

*Expected Utility Theory* yield similar predictions is no surprise: both rest on the behavioral assumption that individuals distort probabilities in order to assess the weight of likelihood associated with random events to which they are faced<sup>5</sup>.

Two other papers consider the case for ambiguity in the knowledge of the probability of accident. In Teitelbaum (2007), the injurer's beliefs are represented by a *neo-additive capacity* (Chateauneuf and alii (2003)): the functional representing his preferences is thus defined as the weighted sum of the best, worst and expected outcomes associated with the ambiguous prospect he faces. In the two-states model, this implies that the injurer under estimates the probability of the accidental event. Then, Teitelbaum shows that neither strict liability nor negligence is generally efficient in the presence of ambiguity. More generally, he finds that the injurer's level of care decreases (increases) with ambiguity if he is optimistic (pessimistic) and decreases (increases) with his degree of optimism (pessimism). However, due to the influence of pessimism, his results suggest once more that negligence may be superior to strict liability in the unilateral accident context. Finally, Franzoni (2012) also considers the case for an ambiguous risk (smooth ambiguity model, coming from the existence of alternative distributions on the probability of accidents), and analyzes both the cases with unilateral and bilateral accidents. He shows that as ambiguity increases the optimal damages also increases under strict liability, while the standard of care is raised under negligence, but only in situations where investing in care has the power to reduce the perceived ambiguity. Moreover, Franzoni show that strict liability dominates negligence but only in very restrictive conditions based on several assumptions: it must be that the injurer has both a lower degree of risk aversion and a lower degree of ambiguity aversion, than the victim, and that the injurer's assessment of the likelihood of harm is less ambiguous<sup>6</sup>.

A common feature of these papers is that they rely on the usual definition of the first best level of care, as captured by condition (1) – this last one only depends on the characteristics of the technology of safety which is available in the economy. In other words, although individuals (both tortfeasors and victims) are assumed to have a subjective assessment and maybe an ambiguous information about the risk of accident, the social planner is supposed to ignore this when it proceeds to the choice of the first best care; moreover, the problem of imperfect risk sharing is not addressed. In the view of a benevolent planner, this may be uneasy to justify<sup>7</sup>.

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<sup>5</sup>Wakker and Tversky (1993) developed the *Cumulative Prospect Theory* which encompasses both models as special cases; it states that there exist two paired functions, a probability function and a utility function on outcomes, defined on the one hand for gains (positive outcomes) and on the other for losses (negative outcomes). Specifically, the shape of the probability function looks like the one in graph 1.

<sup>6</sup>Both the neo-capacity model and the smooth ambiguity model allow to solve the *Ellsberg paradox*. Their main weakness is that, when the ambiguity vanishes the decision context becomes risky, *i.e.* when the knowledge of the true probability is perfect, the model reduce to the Expected Utility one: the individual does not distort the probability of accident. Typically, there exist cumulative evidence that this is not true (see *Allais's paradox*): people distort objectively known probabilities. A classical presentation is Machina (1987).

<sup>7</sup>Typically, the existence of individual bias in risks perception, or the existence of ambiguity in information, are also a major concern for the implementation of public policies in the area

This is the issue of the present paper. We develop the analysis of prevention and tort law when the choice of risk sharing and safety expenditures are no longer separable. We focus on the unilateral accident model in an economy having three main features: 1/ individuals (both injurers and victims) are Rank Dependant Expected Utility maximizers, which allows us to capture two important behavioral characteristics in risk, both pessimism (probability transformation) and risk aversion, which are mainly documented in experimental works; 2/ there exists an aggregate risk in case of accident which entails monetary losses which can not be perfectly compensated, which seems to fit well with the occurrence of accidents leading to large/catastrophic losses; 3/ finally, tortfeasors have the opportunity to invest in damages reduction activities having a monetary cost of effort (hence assuming the perfect substitutability between the cost of effort and wealth) which is a good approximation of the cases where the available technology enables to monitor more precisely the consequences of the accident (losses) than the likelihood of the accident. Thus, our framework departs from previous literature, which considers economies with pure individual risks and perfectly compensable losses. We also add to the literature on questions still in debate: efficient risk sharing rules with endogenous transaction costs (Borch (1962), Raviv (1978)); the relationship between safety standards and wealth (Arlen (1992), Miceli and Segerson (1995), Shavell (1982)); the separation between incentives to prevention, risk sharing and redistributive objectives (Shavell (1981, 1982), Kaplow and Shavell (1994, 2000)); the equivalence between, and efficiency of, basic liability rules (Brown (1973), Landes and Posner (1987), Shavell (1987)).

We characterize the first best of this economy, both in terms of safety expenditures and risk sharing. Specifically, we show that the first best level of care is higher than in a risk-neutral economy, reflecting the existence of an aggregate (non diversifiable) risk which has to be redistributed among society (and thus, shared between injurers and victims) – hence, the first best allocation of risk is such that it is generally not efficient that one party obtains full coverage against the aggregate risk. Moreover, it appears that socially efficient expenditures in safety depend on the aggregate wealth of the economy, which is in contrast with canonical results obtained in the risk neutral/perfectly insurable risk case – however the specific sign of this relationship is shown to be ambiguous. We also show that society’s pessimism has an ambiguous effect on care expenditures; the exception is when society’s preferences are immune against variations of wealth (*i.e.* under constant marginal utility), since more pessimism always yields higher safety expenditures.

The paper also compares the Rank Dependant Expected Utility model with alternative models of preferences. In particular, we show the conditions under which a RDEU economy has expenditures in safety higher than a EU economy. We also give weak conditions regarding the likelihood on the states of the nature, for which EU and RDEU economies have the same first best level of care and

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of safety, health and/or environment regulations. See the classical textbook by Viscusi and ali (2000) for practical issues. Etner and ali (2007), Jeleva and Rossignol (2009) and Salanié and Treich (2009) consider the role of the regulator’s preferences and/or the influence of the political process.

the same risk sharing rules. This suggests that the way preferences and risk attitude are captured may not matter so much. Indeed, our paper insists on the fact that the main issue is the existence of an aggregate risk or not, since it implies that the choice of the care level is no more independent of the risk sharing arrangements. Moreover, we show that (an increase in) pessimism has an ambiguous effect on the first best expenditures in safety.

Turning to the functioning of tort law and liability rules, we find that both strict liability and negligence are no more equivalent under risk aversion, and generally both fail in implementing the first best. The reason is that reaching the first best requires enough instruments in order to transfer wealth in each states, and to set the care level to the due standard; in contrast, a liability rule is associated at best to a due care and to a transfer in the unique event that the accident occurs. As a result, strict liability cannot implement the first best, whether compensatory damages are awarded or not to the victim: as the associated risk sharing is inefficient, offender does not chose an efficient care. Negligence may also fail to induce compliance even when the due care is set to its first best level, given the reverse distortion in risk allocation (the victim retains now the full burden of the damage).

In section 2, we present our framework, and characterize the properties of the first best for a RDEU economy when an aggregate risk exists. Section 3 compares different models of decision under risk and studies the role of pessimism. Section 4 discusses the functioning of liability rules. Section 5 concludes.

## 2 Efficient risk sharing rules and safety activity

### 2.1 A simple model

We consider a simple society with two different groups of identical individuals, injurers and victims, who are initially endowed with wealth  $w = w_0$  and  $y = y_0$ , respectively. Note that  $W_0 = w_0 + y_0$  represents society's initial aggregate wealth—henceforth simply *society's wealth*. An injurer's activity may result in an accident with an exogenous probability  $p > 0$ ; if an accident occurs, a victim suffers a pecuniary loss  $h(x)$ , which depends on the injurer's pecuniary investment in care  $x$ , with  $h'(x) < 0$ ,  $h''(x) > 0$ ,  $h(0) = H > 0$ , and  $h'(\infty) \rightarrow 0$ . Note that we assume that it is always profitable both for the injurer and for society that the injurer undertakes such an activity. However, our analysis does not address questions concerning the optimal level of activity.

We employ a RDEU representation of individuals' preferences (see appendix 1 for a general presentation). We assume that both have the same probability transformation function, denoted  $\varphi : [0, 1] \times [0, 1] \rightarrow [0, 1]$  unique, continuous, increasing and convex in  $p$ , with  $\varphi(0) = 0$  and  $\varphi(1) = 1$ . In contrast, we assume that they are characterized by a specific utility index;  $u(w_i)$  and  $v(y_i)$  will denote the injurer's and the victim's utility in state  $i$ , which are functions of their respective wealth, with  $u', v' > 0$  and  $u'', v'' \leq 0$  – and  $i = b$  in the

accident state (the "bad" state) and  $i = g$  in the no-accident state (the "good" state). Note that according to the assumption made on the various functions ( $\varphi$  convex and  $u$  and  $v$  are both concave), both individuals are risk averse in the strong sense, *i.e.* to second dominance order shifts in risk (Chew, Karni and Saffra (1987)).

Note that as compared to the shape of the probability transformation in graph 1, we focus on the case where individuals always distort probabilities in a pessimistic way, *i.e.*  $\varphi(p) < p \forall p \in ]0, 1[$ . The main consequence of such an assumption is that they have a subjective assessment of the likelihood of accident which is larger than the true (objective, as given by statistical estimates) probability:  $p < 1 - \varphi(1 - p)$ .

We will first consider the problem of a benevolent social planner, which has to choose a certain level of care  $x$  and a certain allocation of risk  $(w_b, w_g; y_b, y_g)$ . The planner's objective is to maximize social welfare, defined as follows:

$$SW = \begin{cases} (1 - \varphi(1 - p)) [u(w_b) + v(y_b)] + \varphi(1 - p) [u(w_g) + v(y_g)] & \text{if } w_b < w_g, y_b < y_g \\ \varphi(p) [u(w_b) + v(y_b)] + (1 - \varphi(p)) [u(w_g) + v(y_g)] & \text{if } w_b > w_g, y_b > y_g \end{cases} \quad (2)$$

subject to the resource constraints<sup>8</sup>:

$$\begin{aligned} w_b + y_b &= w_0 + y_0 - h(x) - x, \text{ in the bad state} \\ w_g + y_g &= w_0 + y_0 - x, \text{ in the good state} \end{aligned}$$

## 2.2 A general characterization of the first-best

Basically, our model illustrates that in situations where an aggregate risk exists, the separation between the allocation of risk and care does not hold; thus the first best is characterized by a certain level of care  $x$  and a certain sharing of the aggregate risk in each state  $(w_b, w_g; y_b, y_g)$  which maximize (2). In this section, we examine the characteristics of the first best.

Accidents reduce the aggregate wealth of society. However, this loss can in principle be allocated in many different ways between the parties involved. The following proposition puts some restrictions on such feasible allocations of risk, and characterizes the optimal level of care it is associated with.

**Proposition 1** *The first-best allocation of risk  $[(w_b, w_g); (y_b, y_g)]$  is comonotonic:  $w_b \leq w_g$  and  $y_b \leq y_g$ , and satisfies Borch's conditions,  $u'(w_b) = v'(y_b)$  and*

<sup>8</sup>Note that in our analysis, we do not retain the aggregate constraint:  $p(w_b + y_b) + (1 - p)(w_g + y_g) = w_0 + y_0 - ph(x) - x$ : this one fits typically a situation with pure idiosyncratic risks (thus, allowing full mutualization), for which the aggregate wealth of the economy is constant over the states of the world (*i.e.* applying the law of large numbers, the constant probability of accident is close to the proportion on the population injured in each state). In such a world, it is well known (Magill and Quinzii (1996)) that (complete) contingent markets may be replaced by a perfect insurance market which pays full insurance to all individuals, and charges a premium equal to the expected indemnity. This is the way Shavell (1982) proceeds. In contrast, we focus on a situation where there is an aggregate risk: social wealth depends on the state of the nature



$u'(w_g) = v'(y_g)$ . The associated first-best level of care satisfies the condition:

$$-(1 - \varphi(1 - p))h'(x^*) + \varphi(1 - p) \left( 1 - \frac{v'(y_g)}{v'(y_b)} \right) = 1 \quad (3)$$

*Proof.* See appendix 2.

In the literature on risk sharing, the property of *comonotonicity* is also termed the *Mutuality Principle*. Its first statements are due to Borch (1962) and Arrow (1964) in the specific case of insurance arrangements (including the administrative costs of insurance contracts). Landsberger and Meilijson (1994), and Chateauneuf, Dana and Tallon (2000) provide additional insights for economies without transaction costs (expenditures needed for the sharing of aggregate resources of society) but when individuals are *Non Bayesian* (Non Expected Utility) maximizers. Thus our result shows that the principle also applies when costly prevention activities are considered. Note, however, that to the extent that we consider only an interior solution, not all comonotonic allocations are efficient – but only those comonotonic allocations that satisfy Borch’s conditions. Such conditions yield that an efficient allocation of risk is reached when, in each state, the aggregate social wealth is shared so that the injurer’s marginal utility of wealth equals the victim’s marginal utility of wealth.

According to (3), the efficient level of care  $x^*$  is such that the planner weighs the victim’s subjective expected (marginal) benefit coming from the loss reduction and the marginal cost of care borne by the injurer (RHS). Note that the RHS in (3), corresponding to the social marginal benefit of safety is defined as the weighted sum of two terms: the decrease in damage  $-h'(x^*)$  and the decrease in the cost of risk  $1 - \frac{v'(y_g)}{v'(y_b)}$ . Those terms are weighted by the likelihood respectively of the bad state  $(1 - \varphi(1 - p))$ , and of the good one  $\varphi(1 - p)$ . Thus, the optimal level of care is now set according to the decrease in the expected damage, plus the decrease in the expected cost of risk, both as they are subjectively assessed/perceived the society. This means that, in contrast to the usual case associated with condition (1), now the optimal level of care is not solely defined according to the characteristics of the technology of safety or the distribution of damage – it must also reflect the preferences of the society (attitude towards risk) and the available opportunities of risk sharing. In order to approach these issues, let us begin with some simple results.

It is important to note that other general features of the first best also hold here, which are in contrast to the risk-neutral world described in (1):

- neither the injurer nor the victim obtains full insurance, that is, neither of them obtains the same wealth in the bad state as in the good state; moreover, in relation to the parties’ utility, neither the injurer nor the victim is protected against adverse changes in his utility, that is the utility is necessarily less in the bad state than in the good state.

- care expenditures are a cost both in the good state and in the bad state, while reducing the magnitude of the loss in the bad state only. Thus, care entails

an implicit transfer from the good state to the bad state, such that at optimum the total cost of accident (care + damage) decreases<sup>9</sup>.

Since conditions (1) and (3) are quite different generally speaking (hence the associated levels of care are different), an important question arises here: how risk aversion affects the efficient level of care<sup>10</sup>?

**Corollary 2** *The first-best level of care when society is risk averse is greater than when society is risk-neutral:  $x^* > \hat{x}$ .*

**Proof.** Risk neutrality under the RDEU representation of preferences requires that  $\varphi(p) = p \forall p$  and at the same time  $v''(y) = u''(w) = 0 \forall y, w$ . Now, note that according to proposition 1:  $y_g \geq y_b$ ; thus, the concavity of  $v$  implies  $v'(y_g) \leq v'(y_b)$ , and thus,  $\varphi(1-p) \left(1 - \frac{v'(y_g)}{v'(y_b)}\right) > 0$  at optimum. Moreover, since by convexity of  $\varphi$  we have:  $\varphi(1-p) < 1-p \Rightarrow 1 - \varphi(1-p) > p$ , it comes that  $-(1 - \varphi(1-p))h'(x) > -ph'(x)$ . As a result, the LHS in (3) satisfies  $-(1 - \varphi(1-p))h'(x) + \varphi(1-p) \left(1 - \frac{v'(y_g)}{v'(y_b)}\right) > -ph'(x)$ , which allows to compare (1) and (3). Given that by convexity of  $h$ , the LHS in (3) decreases in  $x$ , we obtain  $x^* > \hat{x}$ . ■

Condition (3) means that the optimal level of care  $x^*$  minimizes the cost of accident adjusted for the risk; this implies that the first-best level of care  $x^*$  for a RDEU utility-based economy is greater than the first-best level of care  $\hat{x}$  in a risk-neutral economy.

### 2.3 Properties of the first best: comparative statics

The main consequence of our previous findings is that the choice of an efficient level of safety (prevention activity) and the choice of the efficient allocation of risk (allocation of the consequences of the accidents) are inter-related, once we recognize the existence of the aggregate risk, and the limited opportunities to reallocate it among the society.

We study here some more specific properties of the social optimum: how it relates to the aggregate wealth and to the occurrence of accidents<sup>11</sup>.

<sup>9</sup>In appendix 2, we show that (3) also writes

$$h'(x^*) + 1 = -\frac{\varphi(1-p)}{1 - \varphi(1-p)} \frac{v'(y_g)}{v'(y_b)}$$

thus at optimum:  $h'(x) + 1 < 0$  *i.e.* the total cost of accident decreases.

<sup>10</sup>In a related paper (Dari-Mattiacci and Langlais (2012)), we proved that when accidents entail some non monetary losses (which are not insurable), there is no simple answer to such question: state-dependent risk aversion may lead to a higher or a lower level of care than in a risk neutral world. In contrast, our result regarding the influence of risk aversion *per se* is non ambiguous here.

<sup>11</sup>The reader will observe that our results regarding the comparative statics of care having the properties of a *self-insurance* activity are quite simple, and generally, easy to sign. This is in contrast to the analysis of *self-protection* activities, which generally yields ambiguous results

### 2.3.1 The role of society's wealth

From an intuitive (naïve ?) point of view, it seems obvious that the first-best level of care increases when society's wealth increases – thus rich societies would have the opportunity to invest more in safety activities than poorer ones, and be better off this way since the consequences of accidents (damages) are reduced. The next result shows that things are less clear than it seems at first glance:

**Proposition 3** *Consider any first best in terms of safety activity and risk sharing. Then:*

*i) If both individuals have constant absolute risk aversion (CARA), then the first-best level of care is independent from society's wealth.*

*ii) If both individuals have decreasing absolute risk aversion (DARA), then the first-best level of care decreases in society's wealth.*

*iii) If both individuals have increasing absolute risk aversion (IARA), then the first-best level of care increases in society's wealth.*

*Proof.* See appendix 3.

Note that we obtain a result which is in line with more commonplace analysis pertaining to insurance economics or more generally to decision making under risk, since it is well known in such literatures that wealth effects are governed at the individual level through the dependence of the index of risk aversion to the individual wealth. This property extends here to (efficient) collective decisions: the first-best level of care depends on society's wealth; however, the direction of this relation in turn depends on society's risk aversion. Thus, for richer societies it might be optimal to take more or less care than poorer societies. The intuitive explanation is as follows. As we previously observed, the technology of care allows to implement an implicit transfer of wealth from the good state to the bad state in such a risk averse and state-dependant world. According to proposition 5, wealth should be transferred to the state where (initially) society's risk aversion is the smaller; thus, increasing care is optimal if society's risk aversion is smaller in the bad state and *vice versa*, reversing the transfer, when the opposite applies.<sup>12</sup>

An important qualification of proposition 4, is that we focus on the effects of a variation of society aggregate wealth. In contrast, it is easy to see that any

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or much more uneasy to interpret; see the literature in the individual context: Chiu (2000), Jullien and ali (1999), Lee (1998, 2005), Sweeney and Beard (1992). See also Dari-Mattiacci and Langlais (2012) for an application to tort law.

<sup>12</sup>Note that since Borch (1962), it is more usual in the literature on risk sharing to describe wealth effects in terms of *absolute tolerance towards risk* which the inverse of the absolute risk aversion (see appendix 2). The interpretation relating on individual tolerance indexes would be more troublesome. To see this, let us denote  $T_i$  the aggregate index of risk tolerance and  $A_i$  the aggregate index of risk aversion in any state  $i$ ; then straightforward manipulations show that we have:  $T_i/A_i = t_i^u \times t_i^v$ . In words although at the individual level, risk aversion and risk tolerance are inversely related, in contrast at the aggregate level, things are less clear. This explains that for a society to display more risk tolerance in a state, a sufficient (but not necessary) condition is that all individuals be less risk averse in that state.

shift in the initial distribution of wealth between agents, although the aggregate wealth stays constant, has no effect on the efficient level of care, and no effect on the sharing of risk. Consider two economies differing only with regards to the distribution of individual wealths, namely:  $(w_0, y_0)$  and  $(w'_0, y'_0)$  but such that  $w_0 + y_0 = W_0 = w'_0 + y'_0$ . The first best for both economies also satisfies (3) and Borch conditions. Hence, they must have the same efficient expenditure in care, and must adopt the same risk sharing rules. In words, purely redistributive changes in individuals' initial wealth that keep the initial aggregate wealth constant do not affect neither care, nor the efficient allocation of risk.

### 2.3.2 The influence of the probability of accidents

The second crucial parameter of the model is the baseline risk, which is represented by the probability of accident  $p$ . An increase in  $p$  represents an increase in risk borne by the society, in the sense of the First Stochastic Dominance – the new distribution of the fatal event puts more weight of likelihood on the worst state and is also more unfavorable in expected terms (the expected damage increases), all else held equal. The issue is: what is the impact on the level of care, and on the allocation of the various costs of the accident among the society? We prove the following results:

**Proposition 4** *An increase in the probability of accidents leads to an increase in the first-best level of care, an increase in both individuals' wealth in the bad state, and a decrease in both individuals' wealth in the good state.*

*Proof.* See appendix 4.

The intuition for this result is as follows: an increase in  $p$  implies that the bad state becomes relatively more probable than the good state. Thus, it is optimal to transfer some wealth from the good to the bad state. This result can be achieved indirectly, by increasing the level of care, as we have already remarked. More specifically, we have shown that the total accident cost decreases at the first-best level of care. As a result, when care increases, the total wealth in the bad state increases, while obviously the total wealth in the good state decreases due to the investment in care. Due to the mutuality principle, since society is richer in the bad state, so will be both individuals, and vice versa in the good state.

## 2.4 RDEU versus alternative models of choice

In this paragraph, we study the robustness of our results to the relaxation of the assumption regarding the preferences of the parties to the accident.

### 2.4.1 back to the Expected Utility assumption

Let us compare our situation with risk averse RDEU individuals to a society with risk averse Expected Utility individuals. To this end, assume that  $\varphi(p) = p \forall p \in [0, 1]$ , and that the injurer's and the victim's utility are still described by  $u$  and  $v$ .

As a result, we can apply propositions 1 to 7 for the EU economy. The main change is coming from the level of care in the EU economy, which may be characterized for example through the analogue to condition (3):

$$h'(\bar{x}) + 1 = -\frac{1-p}{p} \frac{v'(y_g)}{v'(y_b)} \quad (4)$$

Remark that (3) may be also written as:

$$h'(x^*) + 1 = -\frac{\varphi(1-p)}{1-\varphi(1-p)} \frac{v'(y_g)}{v'(y_b)} \quad (3\text{bis})$$

Notice that  $(p, 1-p)$  is a public information in both society. Thus, it is easy to see that all else equal<sup>13</sup>, the first best level of care is higher in the RDEU economy than in the EU one. Comparing (3bis) and (4), it is clear that by convexity of  $\varphi$  we have  $\varphi(1-p) < 1-p$  and  $\frac{1}{1-\varphi(1-p)} < \frac{1}{p}$  implying that  $\frac{p}{1-p} > \frac{1-\varphi(1-p)}{\varphi(1-p)}$  which is equivalent to  $-\frac{p}{1-p} < -\frac{1-\varphi(1-p)}{\varphi(1-p)}$ . Thus, comparing (3bis) and (4) – the LHS  $h'(x) + 1$  increases with  $x$  – we obtain that the Pareto efficient levels of care satisfy:  $\bar{x} < x^*$ .

The next result show that, under quite simple conditions, both economies are associated with the same optimal allocation of wealth and the same level of care:

**Proposition 5** *Consider a EU economy where the injurer's and the victim's utility are described by  $u$  and  $v$ , both agents having the same probability distribution over the states  $(q, 1-q)$ ; consider a RDEU economy where the injurer's and the victim's utility are described by  $u$  and  $v$ , both agents having the same probability transformation function  $\varphi$  and the same probability distribution over the states  $(p, 1-p)$ . Assuming  $\frac{q}{1-q} = \frac{1-\varphi(1-p)}{\varphi(1-p)}$ , any feasible allocation of care and wealth  $(x, w_b, w_g, y_b, y_g)$  is first best efficient in the EU economy if and only if it is first best efficient in the RDEU economy.*

<sup>13</sup>Nevertheless, we have to pay attention to the next point: a EU economy (with agents having the same probability distribution over the states) and a RDEU economy (agents having the same probability transformation function and the same probability distribution over the states) have the same set of Pareto efficient allocations of wealth; a formal proof of this statement may be found in Chateauneuf, Dana and Tallon (2000, proposition 3.1, corollary 3.2 and proposition 4.2) for an economy without care. It is straightforward that the result extends to economies with care, as shown in the proof of proposition 1. This is not to say that there exist a one to one correspondance between both economies - more specifically, they have the same set of Pareto Optima, which is independant of their belief on the state of the world. More generally, it is well known for EU economies without care, and where individuals have identical priors over the states of nature, that the set of Pareto Optimal allocations of risk is independent of the priors (Chateauneuf and ali (2000), Magill and Quinzii (1996)).

As a result, the specific assumption regarding the representation of (state-independent) preferences may not matter so much: for any RDEU representation, there always exists an EU representation characterized by the same efficient allocation of risk and the same first-best level of risk reduction activity. The same proposition also holds in the context introduced by Bigus (2006), Eide (2007), Franzoni (2012) and Teitelbaum (2006): as long as the same general structures of the economy are maintained (but these papers only consider the case for a small risk of accident), the issue of preferences representation may not be qualitatively so important.

However, the economies may display some differences, depending on the specific assumption required for preferences. We develop an example in appendix 5. Generally speaking, the main differences between the RDEU and EU cases reflect the different effects that an increase in risk or changes of preferences may have on behaviors. The next discussion show that things may become more dramatic, less clearcut, and very connected to the specific assumption regarding the preferences.

#### 2.4.2 the role of pessimism

We highlight here the specific role of the pessimistic transformation of probability captured by the RDEU representation of preferences.

Remark that assuming a constant marginal utility in wealth for both individuals, but maintaining the assumption that both are pessimistic ( $\varphi(p)$  convex  $\forall p \in [0, 1]$ ), we obtain the specific case of the RDEU model termed as the *Dual Theory* suggested by Yaari (1987). Interestingly, the convexity of  $\varphi$  is equivalent to the risk aversion assumption (Roël (1987), Yaari (1987)). A major implication of this assumption is the following:

**Corollary 6** *If both parties have identical preferences, which satisfy the axiomatics of the Dual Theory, any comonotonic allocation of wealth is first best efficient, and the optimal level of care is larger than in the risk-neutral case. Moreover, it increases with society' pessimism.*

**Proof.** The solution to the problem of risk sharing is now undetermined, i.e. it has a infinity of solutions – since both individual are not sensible to marginal changes in wealth (marginal utility in wealth is constant) and both have the same pessimistic assessment of the probability of accident, this implies that any *comonotonic* allocation of the aggregate wealth (aggregate risk) may be implemented. Using that  $v'(y_g) = constant = v'(y_b)$ , this implies that according to (3) the efficient care level satisfies now:

$$-(1 - \varphi(1 - p))h'(x_\varphi) = 1$$

Given that  $1 - \varphi(1 - p) > p$ , then it is still true that:  $x_\varphi > \hat{x}$ .

Now, consider two different economies, characterized respectively by the pessimistic transformations  $\varphi$  and  $\psi$ ; assume that  $\psi$  is a positive and convex

transformation of  $\varphi$ : by definition (Roël (1987) and Yaari (1987)),  $\psi$  is more pessimistic than  $\varphi$  (and thus, more risk averse in the sense of the Dual Theory); thus, for all  $p \in [0, 1]$ , we have:  $\varphi(p) > \psi(p)$ . Finally, this implies that the society with  $\psi$  has a care level which satisfies:

$$-(1 - \psi(1 - p))h'(x_\psi) = 1$$

Since  $1 - \varphi(1 - p) < 1 - \psi(1 - p)$ , we obtain that:  $x_\varphi < x_\psi$ . ■

In our framework, a pessimistic attitude leads to behaviors and results which are the opposite of Teitelbaum (2007)'s ones – the standard of care is larger than in the risk-neutral world, and an increase in the index of pessimism increases the level of care. The basic reason is that under the representation based on a neo additive-additive capacity, the perceived (ambiguous and subjective) likelihood of accident is smaller than the true probability – thus, the benefits of the prevention are under estimated; in contrast, under the RDEU representation with a pessimistic transformation function, the subjective believe on the likelihood of accident is larger than the true probability: thus, the benefits of prevention are over estimated as compared to the expected benefits. Finally, as society is more pessimistic, the care level moves away from the risk neutral one – while it becomes closer under the neo additive capacity representation.

However, note that under the general RDEU representation (*i.e.* assuming  $\varphi$  convex and  $u$  concave), an increase in society's pessimism will have an ambiguous effect on care. To see this, let us consider once more an alternative economy with the same features (identical individuals) as before, excepted for the probability transformation which is now described by the function  $\psi$ . It satisfies the same general assumptions as  $\varphi$ , but for all  $p \in [0, 1]$ , we have:  $\varphi(p) > \psi(p)$ . Then, it leads to an efficient level of care  $x^{**}$  which satisfies now:

$$-(1 - \psi(1 - p))h'(x^{**}) + \psi(1 - p) \left( 1 - \frac{v'(y_g)}{v'(y_b)} \right) = 1$$

to be compared to (9). As far as  $\psi(p) < p$  for all  $p \in [0, 1]$ , it comes that  $1 - \psi(1 - p) > 1 - \varphi(1 - p)$  but  $\psi(1 - p) < \varphi(1 - p)$ . This implies that we may obtain  $x^* \gtrless x^{**}$ . The reason is that the two components of the marginal benefit of safety (RHS in (3)) respond in opposite way to the increase in pessimism. When society becomes more more pessimistic, the benefit of care attached to the decrease in damage  $-h'(x^*)$  increases, since the likelihood of the bad state increases: this requires an increase in the level of care; in contrast, the benefit attached to the opportunities of risk sharing decreases, as the likelihood of the good state decreases: this justifies a decreases in the level of care. Generally speaking, the net effect is thus ambiguous.

### 3 Tort law and liability rules

So far, we have studied the choice of care and risk-sharing policies by a benevolent planner, who can directly implement both of them. In the following, we

extend the analysis to consider whether these two objectives can be reached by means of liability rules. In this context of unilateral prevention, we focus on two simple rules; strict liability and negligence.

Before turning to these issues, we discuss the point that the optimal allocation of wealth associated with the optimal level of care, rest on the characterization of an optimal liability rule – although in an implicit way.

### 3.1 optimal care and implicit liability

We previously have established that the first best endowments  $(w_b, y_b, w_g, y_g)$  increase with  $W_0$ : this is a straightforward result of the *Mutuality Principle*. The main issue here is: whom of both individuals should benefit more of this increase in society’s wealth?

Anticipating on the discussion regarding the implementation of liability rules, the question may be framed as follows: as society becomes richer, is it efficient that firm’s liability in the accident increases – in the sense that the firm should borne a higher proportion of the total cost of the accident? This allows us to display that there exist an *implicit liability rule* that emerge from the analysis of the first best decision<sup>14</sup>.

The next proposition affords some conclusions:

**Proposition 7** *Consider any first best in terms of safety activity and risk sharing. Then, the injurer must benefit more (respectively less) than the victim of an increase in the social wealth if he is less (respectively more) risk averse than the victim.*

**Proof.** In the proof of proposition 4, it has been shown that in each state there exists a redistributive effect, since by Borch’s conditions, we also have the following relationships:

$$\begin{aligned} \frac{\partial y_b}{\partial W_0} &= \frac{t_b^v}{t_b^u} \frac{\partial w_b}{\partial W_0} = \frac{\left(-\frac{u''(w_b)}{u'(w_b)}\right)}{\left(-\frac{v''(y_b)}{v'(y_b)}\right)} \frac{\partial w_b}{\partial W_0} \\ \frac{\partial y_g}{\partial W_0} &= \frac{t_g^v}{t_g^u} \frac{\partial w_g}{\partial W_0} = \frac{\left(-\frac{u''(w_g)}{u'(w_g)}\right)}{\left(-\frac{v''(y_g)}{v'(y_g)}\right)} \frac{\partial w_g}{\partial W_0} \end{aligned}$$

where:  $t_g^v = -\frac{v''(y_g)}{v'(y_g)}$ ,  $t_b^v = -\frac{v''(y_b)}{v'(y_b)}$ ,  $t_g^u = -\frac{u''(w_g)}{u'(w_g)}$ ,  $t_b^u = -\frac{u''(w_b)}{u'(w_b)}$  denote the index of absolute tolerance (inverse of the indexes of the absolute risk aversion). This means that the way the sharing of both the cost of the accident and the cost of care depends on the ratio of the victim’s tolerance to risk to the one of the injurer – or equivalently, it depends on the ratio of the victim’s absolute

<sup>14</sup>Nell and Richter (2003) were also interested in characterizing an optimal liability rule; nevertheless, they only consider the opportunity of monetary transfers in the state of accident, and a linear liability rule.



risk aversion index to the one of the injurer. When the society becomes richer it may be socially efficient that the injurer's allocation  $(w_b, w_g)$  increases more than the victim's one  $(y_b, y_g)$  yielding to a situation where the victim bears an increasing share of the total cost of the accident: this is typically what should occur when the victim's index of absolute risk aversion is smaller than the injurer's one in both states:  $\left(-\frac{v''(y_s)}{v'(y_s)}\right) < \left(-\frac{u''(w_s)}{u'(w_s)}\right)$  for  $s = b, g$ . In words, the liability of the injurer decreases in such a case. But under the reverse conditions, it is socially efficient that the victim bears a decreasing share of the total cost of the accident such that the liability of the injurer increases in such a case. ■

According to considerations of fairness, this result may appear as surprising. However, we consider only efficiency, and we are not explicit regarding the specific institutions which may be adopted to implement the first best. The characterization of this last one is obtained in a general (utilitarian) context: the cost of prevention and investments in safety when these ones are interrelated to socially valuable activities, must be collectively spread. In this sense, socially efficient level of care must be set regarding both technological constraints as those coming from the available technologies of prevention, and the willingness to pay for safety of the population which depends on their preferences under risk. The argument relies on a case where the planner has a sufficient number of instruments, and enough degree of freedom in order to implement any redistribution of costs between parties which is seen as desirable. Hence, irrespective of the specific institutions which may be created to reach it, the first best is always attainable.

### 3.2 Strict liability

We now turn to liability rules<sup>15</sup>. An important result of the previous analysis is that the first best requires enough instruments to reallocate wealth across states. However, liability rules allow transfers between the injurer and the victim in the bad state—in the form of damages payments—while ruling out any payment in the good state. Thus, it may be expected that liability falls short of controlling all of the three variables pertaining to risk-sharing and care and hence will not be enough to implement the first best.

Moreover as previously shown, a first best situation in our type of economy is such that no one should obtain full insurance (i.e. constant personal wealth) but in contrast, both parties should bear some risk. Now, define a second best (Pareto-constrained) situation as one in which one party obtains full insurance against accidents (*i.e.* receives the same wealth, irrespective of the state of the world that materializes)<sup>16</sup>. Fairness in the context of tort law, and in case of the unilateral accident, may be a justification that innocent victims do not suffer any reduction in their wealth as a consequence of accidents that they were not

<sup>15</sup>Our previous draft of the paper (Langlais (2010)) scrutinized the impact of insurance in the unilateral accident model.

<sup>16</sup>See also Dari-Mattiacci and Langlais (2012).

in a position to avoid. A strict liability rule with full compensation of damages for the victim allows to implement this second best situation.

More generally, consider strict liability and assume that the injurer pays damages equal to  $\lambda h(x)$  whenever an accident occurs, where  $\lambda > 0$ . With  $\lambda = 1$ , the injurer pays perfectly compensatory damages to the victim—the victim obtains full compensation for his pecuniary losses and, thus, has a constant wealth across states  $y_b = y_g = y_0$ . However, strict liability can also be designed as to allow for supracompensatory damages ( $\lambda > 1$ , such as punitive damages) or infracompensatory ( $\lambda < 1$ ) damages, in which cases the victim receives a state-dependent wealth which is  $y_0 + (\lambda - 1)h(x)$  in the bad state, and  $y_0$  in the good state.

**Proposition 8** *Under strict liability with perfect compensatory damages  $\lambda = 1$ , the injurer chooses a second-best level of care. The associated allocation of risk is also second best. If damages are infra- or supracompensatory, the outcome is neither a first best nor a second best.*

**Proof.** Assume that the liability rule is strict liability:  $\lambda = 1$ . Under this liability rule, the injurer will take care as to maximize:

$$(1 - \varphi(1 - p)) u(w_0 - \lambda h(x) - x) + \varphi(1 - p) u(w_0 - x)$$

Let  $x_\lambda$  denote the injurer's level of care, which satisfies the following first order condition:

$$-(1 - \varphi(1 - p)) \lambda h'(x_\lambda) + \varphi(1 - p) \left( 1 - \frac{u'(w_0 - x_\lambda)}{u'(w_0 - \lambda h(x_\lambda) - x_\lambda)} \right) = 1 \quad (5)$$

Thus, when  $\lambda = 1$  the injurer's choice of care is by definition the second best level,  $x_1$ , which satisfies:

$$-(1 - \varphi(1 - p)) h'(x_1) + \varphi(1 - p) \left( 1 - \frac{u'(w_0 - x_1)}{u'(w_0 - h(x_1) - x_1)} \right) = 1$$

The wealth of the victim is thus the constant allocation  $\bar{y} = y_0$  which provides him with full insurance.

When  $\lambda \neq 1$ ,  $x_\lambda$  does not meet the condition for a second best level of care. Moreover, such a liability rule generally does not reach neither a first best nor a second best allocation of risk. With supracompensatory damages, the victim receives a greater wealth in the bad state than in the good state:  $y_0 + (\lambda - 1)h(x) > y_0 = y_g$ , implying that the associated allocation of risk is not comonotonic, hence it cannot be first-best efficient. In contrast, with infracompensatory damages we have  $y_0 + (\lambda - 1)h(x) < y_0$ ; then, the allocation is comonotonic but it will be only by chance that Borch's conditions are met; moreover care is not set at the first-best level. ■

From this proposition it emerges that increasing or decreasing the damage amount affects both the level of care and the sharing of the risk, bringing the outcome away from the second best (but possibly improving over it), without being able to reach the first best.

### 3.3 Negligence

Under the negligence rule, the injurer pays damages only if negligent, that is if his level of care is below  $X$ . Here the only policy instrument is the due care level  $X$ . In fact, if the injurer abides by the standard of care, he does not pay damages to the victim, thus  $\lambda$  becomes irrelevant as concerns the allocation of risk.

However, the parameter  $\lambda$  is important in respect of the question of incentive compatibility. When the standard of care is set at the level  $X$ , the utility level of the injurer is defined as:

$$U(w_0, x) = \begin{cases} u(w_0 - x) & \text{if } x \geq X \\ (1 - \varphi(1 - p))u(w_0 - \lambda h(x) - x) + \varphi(1 - p)u(w_0 - x) & \text{otherwise} \end{cases} \quad (6)$$

As a result, under the negligence rule, the injurer obtains a sure outcome ( $w_0 - X$ ) if he adheres to the due care standard, and a risky outcome ( $p, w_0 - \lambda h(x) - x; 1 - p, w_0 - x$ ) if he does not. The usual argument also applies here: according to the first line of (8), the injurer has no incentives to choose  $x > X$ . According to the second line of (8), when he does not comply with  $X$ , the injurer chooses the same level of care as under strict liability;  $x_\lambda$  denotes this level of care.

Thus, negligence raises two issues: Will the injurer comply with the due care? Assuming he does, how does the outcome compare with the first and second best? We define a *dual second best*, as one for which the injurer receives the same constant endowment across the states, and thus, the victim bears the full aggregate risk (which is the dual of the second best).

**Proposition 9** *Under the negligence rule with a due care standard  $X$ :*

- i) If  $X \leq x_\lambda$ , then the injurer complies with the due care standard;*
- ii) If  $X > x_\lambda$ , then the injurer complies with the due care standard only if the following condition is satisfied:*

$$\begin{aligned} & (1 - \varphi(1 - p))u(w_0 - \lambda h(x_\lambda) - x_\lambda) + \varphi(1 - p)u(w_0 - x_\lambda) \\ & \leq u(w_0 - X) \end{aligned} \quad (7)$$

- iii) The allocation of risk is generally not first best. If the injurer complies, the allocation of risk is dual second best.*

**Proof.** i) If  $X \leq x_\lambda$ , then:  $u(w_0 - \lambda h(x_\lambda) - x_\lambda) \leq u(w_0 - x_\lambda) \leq u(w_0 - X)$

which implies in turn:

$$\begin{aligned}
& pu(w_0 - \lambda h(x_\lambda) - x_\lambda) + (1 - p)u(w_0 - x_\lambda) \\
\leq & (1 - \varphi(1 - p)) u(w_0 - x_\lambda) + \varphi(1 - p)u(w_0 - x_\lambda) \\
\leq & (1 - \varphi(1 - p)) u(w_0 - X) + \varphi(1 - p)u(w_0 - X) \\
= & u(w_0 - X)
\end{aligned}$$

Thus, the injurer complies with due care.

ii) if  $X > x_\lambda$ , then:

$$\begin{aligned}
& u(w_0 - x_\lambda) \\
= & (1 - \varphi(1 - p)) u(w_0 - x_\lambda) + \varphi(1 - p)u(w_0 - x_\lambda) \\
\geq & (1 - \varphi(1 - p)) u(w_0 - X) + \varphi(1 - p)u(w_0 - X) \\
= & u(w_0 - X)
\end{aligned}$$

but this inequality is not always satisfied. In several cases, the injurer may prefer to be found liable and bear the loss rather than comply with the due care standard.

iii) When the injurer complies, the victim is not compensated for his loss. The injurer only bears the cost of care and does not face any risk. This outcome is the dual of the second best described above, where the victim did not face any risk. ■

According to proposition 9, note that the results also apply when the standard is  $X = x^*$ . However, given the costs allocation associated with the negligence, the outcome in terms of risk sharing can never be first best. But it is easy to see that whatever the standard  $X$ , the injurer complies as far as it entails a risk reduction as compared to not complying. This requirement is obviously met once we have  $X \leq x_\lambda$ ; hence, the first best in term of prevention may be obtained if  $X = x^* \leq x_\lambda$ . In contrast, by setting  $X = x_\lambda$  and  $\lambda = 1$ , the planner can reach for sure the second-best level of care  $X = x_1$ . Concerning the allocation of risk, note the negligence rule implements a second-best allocation of risk where the injurer, rather than the victim, is fully insured. Finally, the level of care that is second best when the injurer is fully insured can be reached provided that the incentive-compatibility conditions set in the proposition above are satisfied.

## 4 Conclusion

Our paper provides an analysis of firms decisions in the area of safety and care activities, in situations characterized by small probability/large (catastrophic) damages, for which it is well known that individuals display behavioral bias in the perception of the risk. It is also well documented that insurance markets may experience serious failures in such cases, and thus markets insurance coverage

may not exist. Thus our discussion about risk perception by firms is not vacuous, but has sound empirical motivations.

More usual justifications for the case of risk-sensitive firms refer to factors such as the existence of liquidity constraints or the risk of bankruptcy and costly financial distress, and non-linear tax systems. In these cases, the argument is a technical one: although the firm utility index is linear with respect to its profit (constant marginal utility), the constraints coming from the limited ability to spread risk or from the tax system, introduce a non linearity (concavity) in its objective - leading to risk averse decisions by the firms. Other reasons are linked to a non-diversified ownership and/or the delegation of control to a risk-averse manager, whose payoff is linked to firm performances (thus, the preferences of the manager are substituted with the preferences of the firm). The motivation that we consider in this paper relies rather on increasing evidence coming from the experimental literature.

Two salient results have been obtained here (assuming a RDEU representation for individuals' preferences). First, we have shown that the choice regarding safety activities corresponds to a level of care higher than in a risk-free/risk-neutral economy. Second, we have proven that ordinary liability rules are generally inefficient; moreover, negligence does not always lead to a better situation than strict liability.

Our results also illustrate that considering attitudes such as pessimism, when large risks exist, is not only important for the analysis of the functioning of tort law, but that the specific definition of pessimism used in the analysis is crucial: in our framework, pessimism leads to results which are the opposite to those of Teitelbaum (2007). A typical extension of our work is to take into account for the heterogeneity in individuals' risk perceptions. Another one is to consider the context of the bilateral accident for highly risky activities.

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## Appendix 1

Assume a decision maker having preferences which satisfy the RDEU axiomatic; then, there exists two functions:

- a probability transformation  $\varphi : [0, 1] \times [0, 1] \rightarrow [0, 1]$  unique, continuous and increasing in  $p$ , with  $\varphi(0) = 0$  and  $\varphi(1) = 1$ ,
- and a utility index  $u$ , increasing (unique up to a affine transformation), such that facing a risky prospect  $X = (x_1, 1 - p; x_2, p)$ , with  $x_1 < x_2$ , then his satisfaction level is :

$$\begin{aligned} V(X) &\equiv (\varphi(\text{Pr } ob(X \geq x_1)) - \varphi(\text{Pr } ob(X > x_1)))v(x_1) \\ &\quad + (\varphi(\text{Pr } ob(X \geq x_2)) - \varphi(\text{Pr } ob(X > x_2)))v(x_2) \\ &\equiv (1 - \varphi(p))v(x_1) + \varphi(p)v(x_2) \end{aligned}$$

It is well known that two different definitions for the concept of risk aversion are both characterized by the concavity of the utility index in the Expected Utility model, namely:

- risk aversion in the sense of Rothschild and Stiglitz – aversion to a mean-preserving spread of risk, which allows to compare two risky situations, differing according to the second stochastic dominance order;
- risk aversion in the sense of Arrow and Pratt – preferring the certainty of the expected gamble to the gamble, we compare certainty to a risky outcome,

In contrast, in the RDEU framework, risk aversion in the sense of Rothschild and Stiglitz is equivalent to  $\varphi$  convex and  $u$  concave (Chew and ali (1990)): this is a concept of *strong* aversion to risk (Cohen (1995)), that is aversion to marginal shifts in risk. In contrast, risk aversion in the sense of Arrow and Pratt does not necessarily require that  $u$  be concave, as far as  $\varphi$  is sufficiently convex (Chateauneuf and Cohen (1994)): this is a concept of *weak* risk aversion, or aversion to a global increase in risk.

In the literature, the convexity of  $\varphi$  is also associated with a behavior termed *probabilistic risk aversion* (litteraly: aversion to probability mixtures) or *strong pessimism* (Roël (1987), Wakker (1994), Yaari (1987)).

The consequence of the convexity of  $\varphi$  together with the conditions  $\varphi(0) = 0$  and  $\varphi(1) = 1$ , is that  $\varphi(q) < q$  for all  $q \in [0, 1]$ . Thus, the way a risk averse decision maker evaluates the prospect  $X$  in the RDEU model implies that  $1 - \varphi(p) > 1 - p$ : he over estimates the probability of the smaller gain  $1 - p$ , and  $\varphi(p) < p$ , he under estimates the probability of the larger one  $p$ .

## Appendix 2

### PROOF OF PROPOSITION 1.

Assume first that there exists some values of  $x > 0$  such that  $h(x) + x < H$ . Assume now that the feasible allocation  $[(w_b, w_g); (y_b, y_g)]$  with  $w_b \leq w_g$  and simultaneously  $y_b \geq y_g$ , associated with a care level  $x$ , is Pareto optimal.

Now for the same level of care, define an alternative feasible allocation  $[(\tilde{w}_b, \tilde{w}_g); (\tilde{y}_b, \tilde{y}_g)]$  where  $\tilde{w}_b \leq \tilde{w}_g$  and simultaneously  $\tilde{y}_b = \tilde{y}_g$ , such that:

$$\begin{aligned}\tilde{w}_b &= w_b + (1-p)(y_b - y_g) \\ \tilde{w}_g &= w_g - p(y_b - y_g) \\ \tilde{y}_b &= py_b + (1-p)y_g = \tilde{y}_g \\ \tilde{w}_b + \tilde{y}_b &= w_b + y_b \\ \tilde{w}_g + \tilde{y}_g &= w_g + y_g\end{aligned}$$

By definition, both individuals obtain the same expected individual wealth irrespective of the allocation we choose, since:  $p\tilde{y}_b + (1-p)\tilde{y}_g = py_b + (1-p)y_g$  for the victim and  $p\tilde{w}_b + (1-p)\tilde{w}_g = pw_b + (1-p)w_g$  for the injurer.

On the other hand,  $(\tilde{w}_b, \tilde{w}_g)$  is less spread than  $(w_b, w_g)$  in the sense of the second stochastic dominance order, since given that for the same probabilities  $(p, 1-p)$  we have the following order for injurer's wealth in the different states:  $w_b < \tilde{w}_b \leq \tilde{w}_g < w_g$ ;  $(\tilde{y}_b, \tilde{y}_g)$  is also less spread than  $(y_b, y_g)$  since we have:  $y_b > \tilde{y}_b = \tilde{y}_g > y_g$ . Recall that, by assumption, both individuals are risk averse to second dominance order shifts in risk. Thus  $[(\tilde{w}_b, \tilde{w}_g); (\tilde{y}_b, \tilde{y}_g)]$  Pareto dominates  $[(w_b, w_g); (y_b, y_g)]$ ; hence a contradiction.

Now, define two real numbers  $\sigma_b$  and  $\sigma_g$  as the shadow prices of the aggregate resource constraints of society. The problem of the social planner is now equivalent to the maximization of:

$$(1 - \varphi(1-p)) [u(w_b) + v(y_b)] + \varphi(1-p) [u(w_g) + v(y_g)]$$

under the resources constraints. When an interior solution exists, then it corresponds to a vector  $(x, w_b, w_g, y_b, y_g)$  which satisfies the set of the following conditions:

$$-\sigma_b h'(x) - (\sigma_g + \sigma_b) = 0 \tag{A}$$

$$(1 - \varphi(1-p))u'(w_b) - \sigma_b = 0 \tag{B}$$

$$\varphi(1-p)u'(w_g) - \sigma_g = 0 \tag{C}$$

$$(1 - \varphi(1-p))v'(y_b) - \sigma_b = 0 \tag{D}$$

$$\varphi(1-p)v'(y_g) - \sigma_g = 0 \tag{E}$$

Conditions (B) to (E) define the rule that should be used by the planner to implement a first best allocation of risk. Using (B) and (C) together, and (D) and (E) together, we obtain Borch's conditions:

$$\begin{aligned}u'(w_b) &= v'(y_b), \text{ with } w_b + y_b = w_0 + y_0 - x - h(x) \\ u'(w_g) &= v'(y_g), \text{ with } w_g + y_g = w_0 + y_0 - x\end{aligned}$$

which leads to:

$$\frac{v'(y_g)}{v'(y_b)} = \frac{u'(w_g)}{u'(w_b)}$$

Now summing over conditions (B) to (E) yields:

$$\sigma_b + \sigma_g = (1 - \varphi(1 - p))v'(y_b) + \varphi(1 - p)v'(y_g)$$

Substituting in (A) and rearranging, we obtain that the first-best level of care satisfies the condition:

$$-(1 - \varphi(1 - p))h'(x^*) + \varphi(1 - p) \left(1 - \frac{v'(y_g)}{v'(y_b)}\right) = 1 \quad (\text{F})$$

Condition (F) is equivalent to:

$$h'(x^*) + 1 = -\frac{\varphi(1 - p)}{1 - \varphi(1 - p)} \frac{v'(y_g)}{v'(y_b)} \quad (\text{I})$$

which allows to show that at optimum:  $h'(x) + 1 < 0$  (since the RHS is negative): *i.e.* the total cost of accident (damage to the victim + cost of care) is decreasing at optimum.

Finally, second order conditions require the following inequality to hold:

$$\frac{h''(x^*)}{1 + h'(x^*)} (t_g^u + t_g^v) + \frac{t_g^u + t_g^v}{t_b^u + t_b^v} (1 + h'(x^*)) - 1 < 0$$

where:  $t_g^v = -\frac{v''(y_g)}{v''(y_b)}$ ,  $t_b^v = -\frac{v''(y_b)}{v''(y_b)}$ ,  $t_g^u = -\frac{u''(w_g)}{u''(w_g)}$ ,  $t_b^u = -\frac{u''(w_b)}{u''(w_b)}$  denote the inverse of the indexes of risk aversion<sup>17</sup> for the victim and the injurer, evaluated for each state; given the various conditions made on preferences and on the technology of safety, this last inequality is obviously satisfied.

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<sup>17</sup>Our terminology is quite abusive. In the RDEU model, the *local* characterization of risk aversion (or its local measure *à la* Arrow-Pratt) do not necessarily require the concavity of the utility index (see Chateauneuf and Cohen (1993), Cohen (1995) and Courtaut and Gayant (1998)). In the sense of Segal and Spivak (1990), the RDEU model displays *first order risk aversion*, due to the convexity of  $\varphi$ ; in contrast, the index  $-\frac{u''}{u'}$  corresponds to *second order risk aversion* (this is a second order term) which is specific to the Expected Utility model. More generally, since Allais it is a matter of debate in decision theory (see Bouyssou and Vansnick (1990)) that the concavity of the utility index represents both the risk aversion assumption (a behavior under risk) *and* the hypothesis of the decreasing marginal utility of a wealth (a feature of preferences with respect to certain outcomes). However, in order to keep easier the interpretation if the results, we make use here of the widely spread terminology, assuming that the ratio  $-\frac{u''}{u'}$  is driving the behavior under risk with respect to the variation of his wealth. Courtaut and Gayant (1998) suggest that in the RDEU model, the ratio  $-\frac{u''}{u'}$  is a measure of *spreading risk aversion*: << the decrease of marginal utility induces mechanically a gap between the certainty equivalent of a random variable and its expectation (Courtaut and Gayant (p 212, 1998)) >>.

## Appendix 3

### PROOF OF PROPOSITION 4.

Society's initial wealth is  $W_0 = w_0 + y_0$ . Given that according to the resources constraints, we have:

$$\begin{aligned}\frac{\partial y_b}{\partial W_0} + \frac{\partial w_b}{\partial W_0} &= 1 - (1 + h'(x)) \frac{\partial x}{\partial W_0} \\ \frac{\partial y_g}{\partial W_0} + \frac{\partial w_g}{\partial W_0} &= 1 - \frac{\partial x}{\partial W_0}\end{aligned}$$

the impact of an increase in  $W_0$  on the individual endowments (risk sharing rules) may be obtained by first totally differentiating Borch's conditions, to obtain:

$$\begin{aligned}u''(w_b) \frac{\partial w_b}{\partial W_0} &= v''(y_b) \frac{\partial y_b}{\partial W_0} \Rightarrow \frac{\partial y_b}{\partial W_0} = \frac{t_b^v}{t_b^u} \frac{\partial w_b}{\partial W_0} \\ u''(w_g) \frac{\partial w_g}{\partial W_0} &= v''(y_g) \frac{\partial y_g}{\partial W_0} \Rightarrow \frac{\partial y_g}{\partial W_0} = \frac{t_g^v}{t_g^u} \frac{\partial w_g}{\partial W_0}\end{aligned}$$

and substituting the results of the derivation of the constraints, we obtain:

$$\frac{\partial y_b}{\partial W_0} = \frac{t_b^v}{t_b^u + t_b^v} \left( 1 - (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0} \right) \quad (\text{J})$$

$$\frac{\partial y_g}{\partial W_0} = \frac{t_g^v}{t_g^u + t_g^v} \left( 1 - \frac{\partial x^*}{\partial W_0} \right) \quad (\text{K})$$

Then, differentiating condition (3) and rearranging, we obtain:

$$\frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x^*}{\partial W_0} = \frac{1}{t_b^v} \frac{\partial y_b}{\partial W_0} - \frac{1}{t_g^v} \frac{\partial y_g}{\partial W_0} \quad (\text{L})$$

After substituting (J) and (K) into (L), we have:

$$\frac{\partial x^*}{\partial W_0} = \frac{\frac{t_g^u + t_g^v}{t_b^u + t_b^v} - 1}{\frac{h''(x^*)}{1 + h'(x^*)} (t_g^u + t_g^v) + \frac{t_g^u + t_g^v}{t_b^u + t_b^v} (h'(x^*) + 1) - 1} \quad (\text{M})$$

where:  $t_g^v = -\frac{v'(y_g)}{v''(y_g)}$ ,  $t_b^v = -\frac{v'(y_b)}{v''(y_b)}$ ,  $t_g^u = -\frac{u'(w_g)}{u''(w_g)}$ ,  $t_b^u = -\frac{u'(w_b)}{u''(w_b)}$  have been defined in the proof of proposition 1. Given that according to the second order condition associated with the first best the denominator has a negative sign, we have:

$$\text{sign} \frac{\partial x^*}{\partial W_0} = \text{sign} \left( 1 - \frac{t_g^u + t_g^v}{t_b^u + t_b^v} \right)$$

Now, remember that according to the Mutuality Principle, the individual endowments satisfy  $w_b \leq w_g$  and  $y_b \leq y_g$ ; hence, the individual indexes of tolerance may be ranked according to the property of the corresponding indexes of absolute risk aversion. Thus, by definition we have:

- under CARA:  $-\frac{v''(y_g)}{v'(y_g)} = -\frac{v''(y_b)}{v'(y_b)}$  and  $-\frac{u''(w_g)}{u'(w_g)} = -\frac{u''(w_b)}{u'(w_b)}$ ; this implies:  
 $t_g^v = t_b^v$  and  $t_g^u = t_b^u \Rightarrow \frac{t_g^u + t_g^v}{t_b^u + t_b^v} = 1 \Rightarrow \frac{\partial x^*}{\partial W_0} = 0$ ;

- under DARA:  $-\frac{v''(y_g)}{v'(y_g)} < -\frac{v''(y_b)}{v'(y_b)}$  and  $-\frac{u''(w_g)}{u'(w_g)} < -\frac{u''(w_b)}{u'(w_b)}$ ; this implies  
now  $t_g^v \geq t_b^v$  and  $t_g^u \geq t_b^u \Rightarrow \frac{t_g^u + t_g^v}{t_b^u + t_b^v} > 1 \Rightarrow \frac{\partial x^*}{\partial W_0} < 0$ ;

- under IARA:  $-\frac{v''(y_g)}{v'(y_g)} > -\frac{v''(y_b)}{v'(y_b)}$  and  $-\frac{u''(w_g)}{u'(w_g)} > -\frac{u''(w_b)}{u'(w_b)}$ ; finally, this  
implies  $t_g^v \leq t_b^v$  and  $t_g^u \leq t_b^u \Rightarrow \frac{t_g^u + t_g^v}{t_b^u + t_b^v} < 1 \Rightarrow \frac{\partial x^*}{\partial W_0} > 0$ .

Hence the results.

## Appendix 4

### PROOF OF PROPOSITION 5.

Given that  $\frac{\partial y_b}{\partial p} + \frac{\partial w_b}{\partial p} = -(1 + h'(x^*))\frac{\partial x^*}{\partial p}$  and  $\frac{\partial y_g}{\partial p} + \frac{\partial w_g}{\partial p} = -\frac{\partial x^*}{\partial p}$ , the impact on the risk sharing rules of any increase in  $p$  may be obtain first totally differentiating Borch's conditions to obtain:

$$\begin{aligned} u''(w_b) \frac{\partial w_b}{\partial p} &= v''(y_b) \frac{\partial y_b}{\partial p} \Rightarrow \frac{\partial y_b}{\partial p} = \frac{t_b^v}{t_b^u} \frac{\partial w_b}{\partial p} \\ u''(w_g) \frac{\partial w_g}{\partial p} &= v''(y_g) \frac{\partial y_g}{\partial p} \Rightarrow \frac{\partial y_g}{\partial p} = \frac{t_g^v}{t_g^u} \frac{\partial w_g}{\partial p} \end{aligned}$$

and once more substituting the constraints we obtain:

$$\left. \begin{aligned} \frac{\partial y_b}{\partial p} &= -\frac{t_b^v}{t_b^u + t_b^v} (1 + h'(x^*)) \frac{\partial x^*}{\partial p} \\ \frac{\partial y_g}{\partial p} &= -\frac{t_g^v}{t_g^u + t_g^v} \frac{\partial x^*}{\partial p} \end{aligned} \right\} \quad (\text{N})$$

Then, differentiating condition (3) and rearranging, we obtain:

$$\frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x^*}{\partial p} - \frac{1}{t_b^v} \frac{\partial y_b}{\partial p} + \frac{1}{t_g^v} \frac{\partial y_g}{\partial p} = -\frac{\varphi'(1-p)}{(1 - \varphi(1-p))\varphi(1-p)} \quad (\text{O})$$

Substituting (N) in (O) gives:

$$\frac{\partial x^*}{\partial p} = -\frac{\left( \frac{\varphi'(1-p)}{(1 - \varphi(1-p))\varphi(1-p)} \right) \times (t_g^u + t_g^v)}{\frac{h''(x^*)}{1 + h'(x^*)} (t_g^u + t_g^v) + \frac{t_g^u + t_g^v}{t_b^u + t_b^v} (1 + h'(x^*)) - 1} > 0 \quad (\text{P})$$

Coming back to (O) and remembering that at optimum we must have  $-(1 + h'(x^*)) > 0$ , it comes that  $\frac{\partial y_b}{\partial p} > 0$  and  $\frac{\partial y_g}{\partial p} < 0$ .

## Appendix 5

The differences between the RDEU and EU cases may be the result for example of the different effects of an increase in risk or changes in the intensity of risk aversion (preferences). For example note first that it is straightforward to verify that both share the same sensitivity to wealth effects all else held equal. In contrast, using (O) and setting  $\varphi(p) = p \forall p \in ]0, 1[$ , it comes that the risk sensitivity in a EU economy verifies:

$$\frac{\partial x^{EU}}{\partial p} = -\frac{\frac{t_g^u + t_g^v}{p(1-p)}}{\frac{h''(x^{EU})}{1+h'(x^{EU})}(t_g^u + t_g^v) + \frac{t_g^u + t_g^v}{t_b^u + t_b^v}(1 + h'(x^{EU})) - 1} > 0$$

Hence, the sensitivity of care to the probability of accident may be higher or smaller in the RDEU economy then in the EU economy. To see this, note that by convexity of  $\varphi$ , it comes that  $\varphi'(1-p) > \frac{\varphi(1-p)}{1-p} \Leftrightarrow \frac{\varphi'(1-p)}{\varphi(1-p)} > \frac{1}{1-p}$ ; but given that  $1 - \varphi(1-p) > p \Leftrightarrow \frac{1}{1-\varphi(1-p)} < \frac{1}{p}$ , we obtain that  $\frac{\varphi'(1-p)}{\varphi(1-p)} \frac{1}{1-\varphi(1-p)} \geq \frac{1}{p(1-p)}$  implying that all else held equal  $\frac{\partial x^*}{\partial p} \geq \frac{\partial x^{EU}}{\partial p}$ .

## Appendix 6

PROOF OF PROPOSITION 9.

Let us define the function  $s(k) \equiv \frac{u'(g+k)}{u'(b+k)}$  where  $g > b$ ; then if the injurer is DARA,  $s(k)$  is increasing, while if the injurer is IARA,  $s(k)$  is decreasing. This is straightforward since:

$$s'(k) \equiv \frac{u'(g+k)}{u'(b+k)} \left[ \left( -\frac{u''(b+k)}{u'(b+k)} \right) - \left( -\frac{u''(g+k)}{u'(g+k)} \right) \right]$$

and thus:  $sign[s'(k)] = sign \left[ \left( -\frac{u''(b+k)}{u'(b+k)} \right) - \left( -\frac{u''(g+k)}{u'(g+k)} \right) \right]$ .

By definition, the first best requires the sharing of the total cost of accident by both parties (each one bears a part of the aggregate risk, according to the Mutuality Principle), and satisfies  $\tilde{y}_b < \tilde{y}_g$  and  $\tilde{w}_b = W_0 - \tilde{y}_b - h(x) - x < \tilde{w}_g = W_0 - \tilde{y}_g - x$ . In contrast, should the planner adheres to moral considerations, the second best corresponds to an allocation where  $y_b = y_g$  but  $w_b = W_0 - \bar{y} - h(x) -$

$x < w_g = W_0 - \bar{y} - x$ . By concavity of  $u$ , we also have:  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \tilde{y}_b - x)}{u'(W_0 - \tilde{y}_b - h(x) - x)}$   
and  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \tilde{y}_g - x)}{u'(W_0 - \tilde{y}_g - h(x) - x)}$ .

Consider two cases:

i) Let us define now as:  $\bar{y} = \tilde{y}_b - k$ . It comes that if the injurer is IARA, we obtain:  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \tilde{y}_b - x)}{u'(W_0 - \tilde{y}_b - h(x) - x)} > \frac{u'(W_0 - \tilde{y}_b + k - x)}{u'(W_0 - \tilde{y}_b + k - h(x) - x)}$ .

Hence, by continuity, for any feasible  $\bar{y} < \tilde{y}_b$  we have under IARA:  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \bar{y} - x)}{u'(W_0 - \bar{y} - h(x) - x)}$ . As a result, it comes that:  $\varphi(1 - p) \left(1 - \frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)}\right) < \varphi(1 - p) \left(1 - \frac{u'(W_0 - \bar{y} - x)}{u'(W_0 - \bar{y} - h(x) - x)}\right)$ ; hence, the LHS in (3) is smaller than the LHS in (5), and finally this implies:  $x^* < x_{\bar{y}}$ .

ii) Let us now define as:  $\bar{y} = \tilde{y}_g + k$ . It comes that if the injurer is DARA, we obtain:  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \tilde{y}_g - x)}{u'(W_0 - \tilde{y}_g - h(x) - x)} > \frac{u'(W_0 - \tilde{y}_g - k - x)}{u'(W_0 - \tilde{y}_g - k - h(x) - x)}$ . Hence, by continuity, for any feasible  $\bar{y} > \tilde{y}_g$  we have under DARA:  $\frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)} > \frac{u'(W_0 - \bar{y} - x)}{u'(W_0 - \bar{y} - h(x) - x)}$ . As a result, we have:  $\varphi(1 - p) \left(1 - \frac{u'(\tilde{w}_g)}{u'(\tilde{w}_b)}\right) < \varphi(1 - p) \left(1 - \frac{u'(W_0 - \bar{y} - x)}{u'(W_0 - \bar{y} - h(x) - x)}\right)$ ; hence, the LHS in (3) is still smaller than the LHS in (5), and finally it comes also that:  $x^* < x_{\bar{y}}$ .